# 中性子星内部磁場の安定性

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アウトライン

０　イントロ

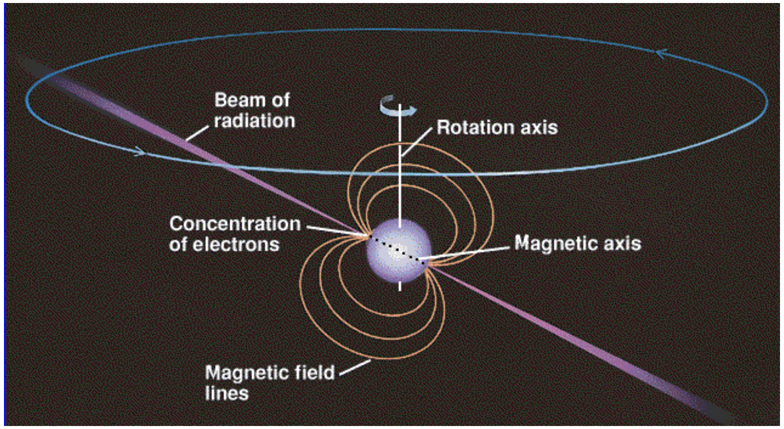
１　磁場中性子星の平衡形状

２　磁場の不安定性（線形解析）

３　シミュレーション結果（軸対称）

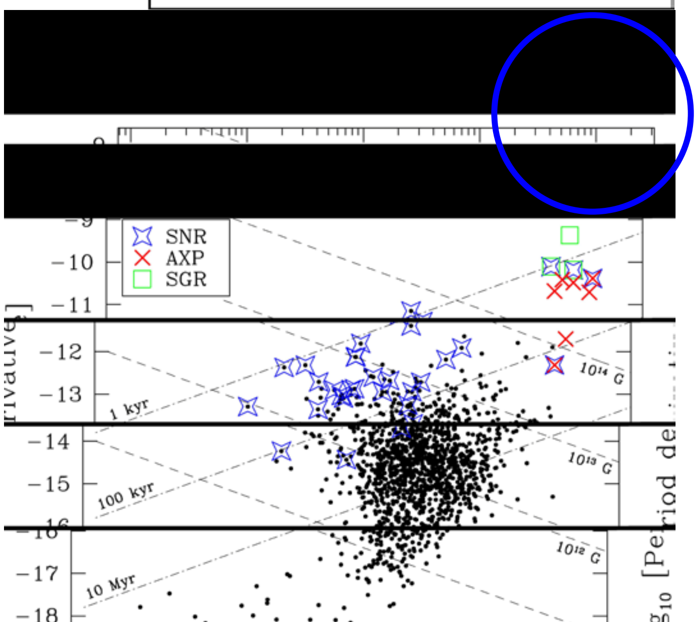
４　シミュレーション結果（非軸対称）

# Introduction




Pulsars are believed to be neutron stars which have strong magnetic field ($B \sim 10^{12}$G)

$P - \dot{P}$ Diagram




There are some speical classes called magnetars which have a very strong magnetic field $B \sim 10^{14-15}$G.

What is an origin of such stong magnetic fields ?

Magnetically driven SN (many references) ? or dynamo process in proto NS ?(Thompson & Duncun 93 etc.).

# Introduction

We don't have information of interior magnetic fields because

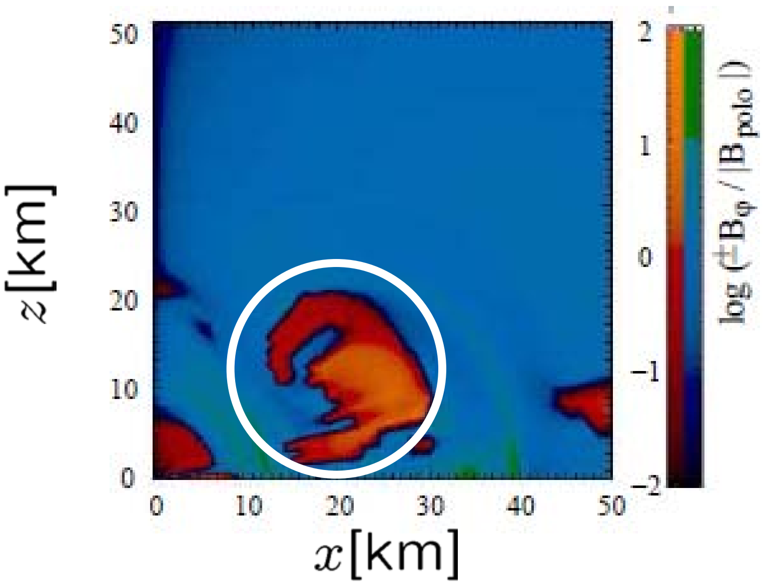
$$\frac{dE}{dt} = \frac{d}{dt}\left(\frac{1}{2}I\Omega^2\right) = -4\pi^2 I \frac{\dot{P}}{P^3}$$

assuming dipole radiation

$$\Rightarrow \quad B_p \propto (IP\dot{P})^{1/2} R^{-3}$$

Note that $10^{14-15}$G is poloidal component.

Toroidal / Poloidal (Cerda Duran et al. (08))




Also, toroidal field is easily generated by magnetic winding.

$$\partial_t B_\varphi = B^\varpi \partial_\varpi \Omega + B^z \partial_z \Omega$$

So, there is a possibility that toroidal field dominates the poloidal component.

# Introduction

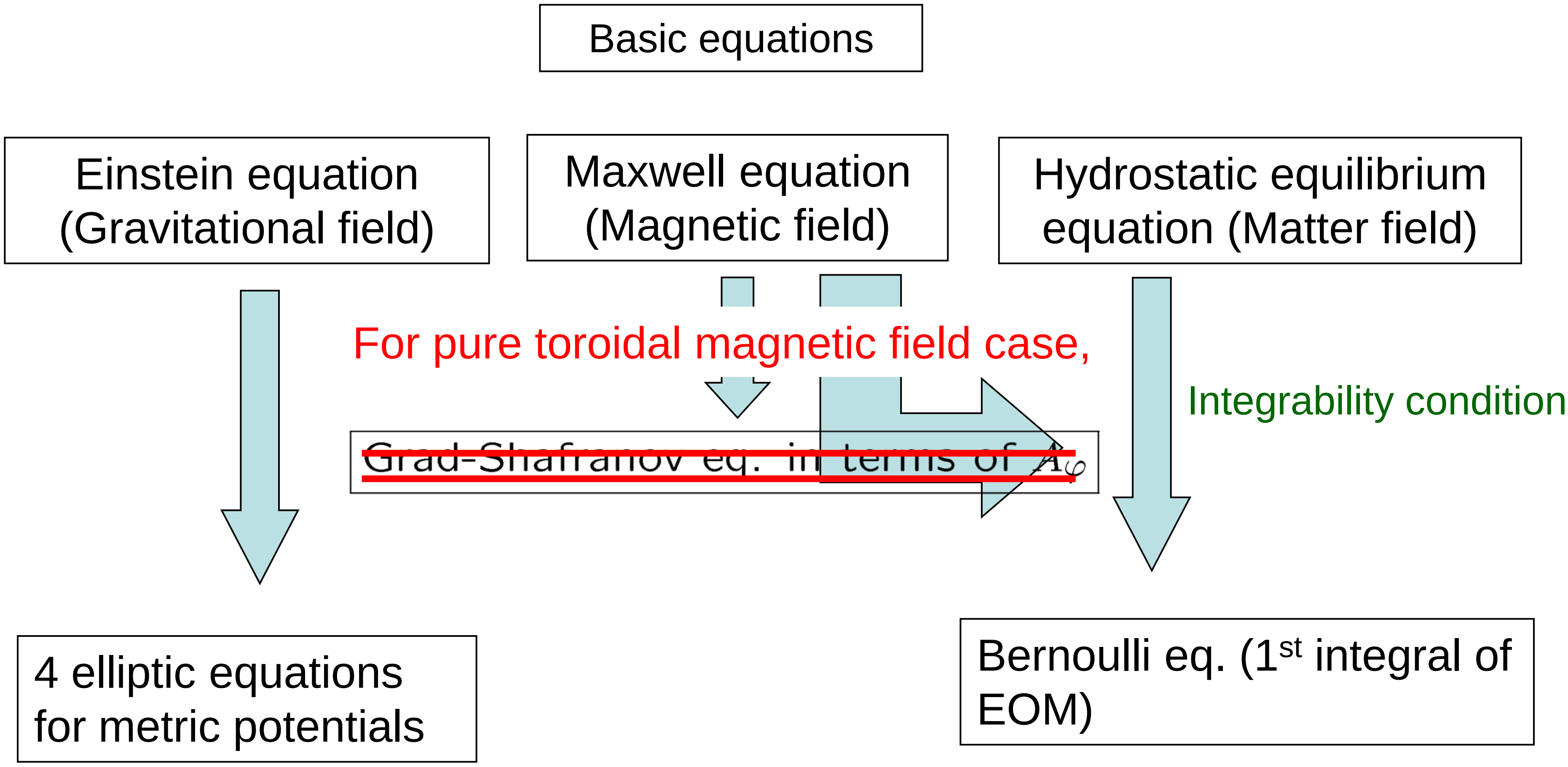
- Toroidally magnetized relativistic star in equilibrium
- Explore the stability of toroidal magnetic field (GRMHD simulation)

# How to construct magnetized star in equilibrium

Basic assumptions

0. Stationary and axisymmetry
1. Perfect fluid
2. No meridional flow
3. Ideal MHD
4. Pure toroidal magnetic field
5. Barotropic EOS

# How to construct magnetized star in equilibrium



Simple analogy for intergrability conditions

Newtonian rotating equilibrium configurations without magnetic field

$$\text{hydrostatic eq.} \quad \frac{\nabla P}{\rho} + \nabla \phi_g + \varpi \Omega^2 \nabla \varpi = 0$$

if $P = P(\rho)$ or $\Omega = \Omega(\varpi)$ (Tassoul 00)

$$\text{Bernoulli eq.} \quad \int \frac{dP}{\rho} + \phi_g + \int \Omega^2 \varpi d\varpi = C$$

Purely toroidal magnetic field in general relativistic equilibrium is given by

$$\text{Faraday tensor} \quad F_{r\theta} = (\text{metric}) \times K(u)$$

where $u = \rho_0 h (r \sin\theta)^2 \times (\text{metric})$.

We simpliy choose $K(u)$ as

$$K(u) = bu^k \text{ with } k \geq 1 \Rightarrow B_{(\varphi)} \propto \rho^k (r \sin\theta)^{2k-1}$$

# How to construct magnetized star in equilibrium

Basic equations

Einstein equation (Gravitational field)

Maxwell equation (Magnetic field)

Hydrostatic equilibrium equation (Matter field)

For pure toroidal magnetic field case,

Integrability condition

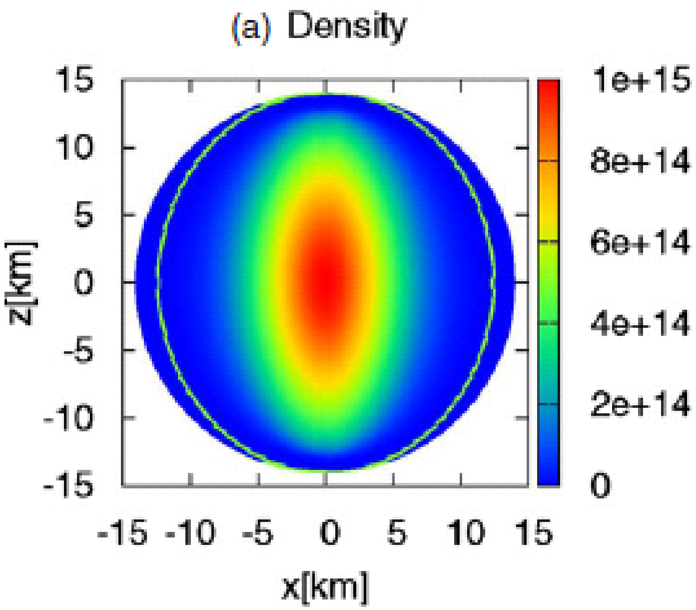
4 elliptic equations for metric potentials

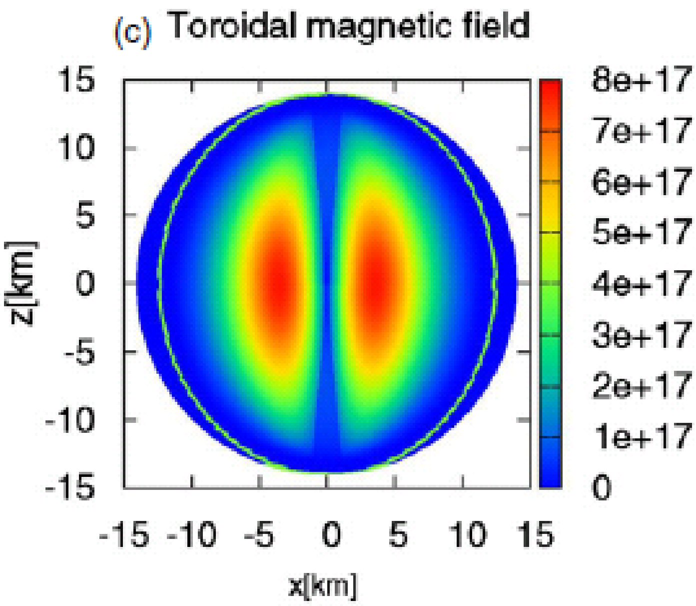
Bernoulli eq. (1st integral of EOM)

Numerical scheme : KEH (Komatsu et al.1987)

# Magnetized Neutron Star

Stellar structure on the meridional plane

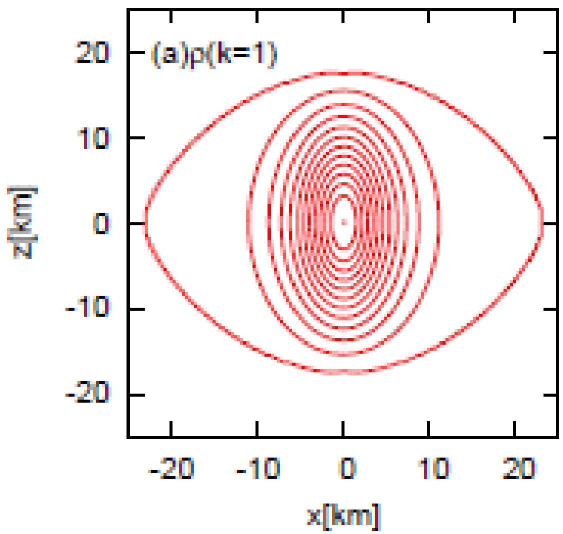


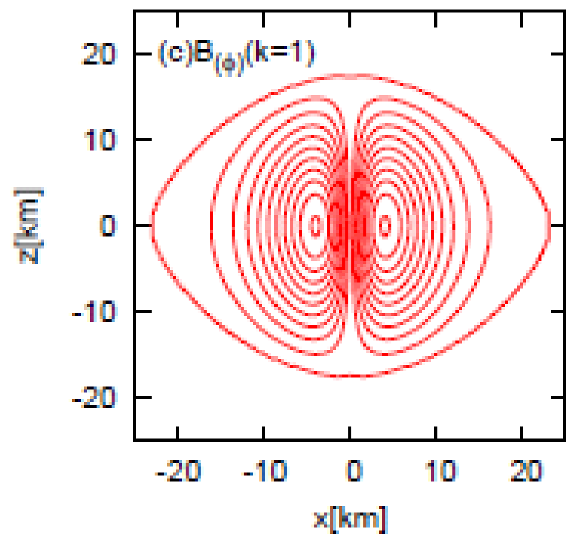

1. Concentration of toroidal field deep inside the star
2. Prolate stellar configuration due to the strong magnetic fields
3. It is possible to construct hyper strong magnetized star, e.g., H/|W|=O(0.1)

# Magnetized Rotating Neutron Star

Density




Magnetic field




- Oblate stellar surface
- Prolate structure deep inside star

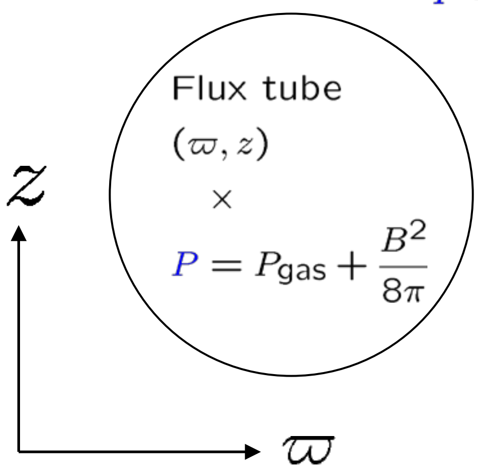
# Stability of toroidal magnetic field

Cylindrical coordinate $(\varpi, z, \varphi)$

**Critical radius** : $\varpi_c \equiv 2c_s^2/g_\varpi$

Surroundings

$P = P_{\rm gas}$




Density deficit of flux tube : $\delta\rho = \frac{B^2}{8\pi c_s^2}$

$$\Downarrow$$

Magnetic buoyancy force : $g\frac{B^2}{8\pi c_s^2}$

Hoop stress : $\frac{B^2}{4\pi\varpi}$

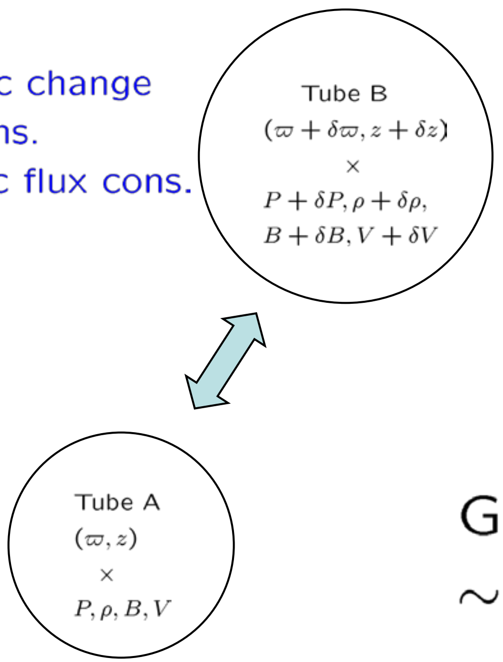
$\varpi < (>) \varpi_c \Leftrightarrow$ Hoop stress $>$ $(<)$Magnetic buoyancy force

# Stability of toroidal magnetic field

- Interchange instability ($m = 0$ mode)
- Tayler instability ($m = 1$ mode, Tayler 73)
- Parker instability ($m \neq 0$ mode, Parker 66)

m=0

∘Adiabatic change
∘Mass cons.
∘Magnetic flux cons.

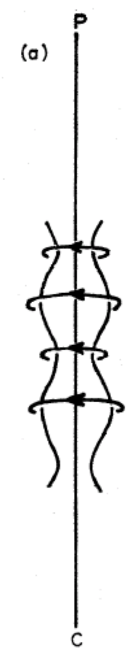



Instability criterion

$$\left(\frac{1}{\varpi} - \frac{1}{\varpi_c}\right) \partial_\varpi \ln \left(\frac{B_{(\varphi)}}{\rho\varpi}\right) > 0$$

$$\frac{g_z}{c_s^2} \partial_z \ln \left(\frac{B_{(\varphi)}}{\rho\varpi}\right) < 0$$

Growth time scale
$\sim$ Alfvén time scale $R/v_A$



# Stability of toroidal magnetic field

- Interchange instability ($m = 0$ mode)
- Tayler instability ($m = 1$ mode, Tayler 73)
- Parker instability ($m \neq 0$ mode, Parker 66)

m=1


Instability criterion

$$\left(\frac{2}{\varpi_c} - \frac{2}{\varpi}\right) \partial_\varpi \ln\left(B_{(\varphi)}\varpi\right) + \frac{m^2}{\varpi^2} < 0$$

$$\frac{g_z}{c_s^2}\partial_z \ln\left(B_{(\varphi)}\varpi\right) + \frac{m^2}{\varpi^2} < 0$$

Mainly focusing on $\varpi \sim 0$

Growth time scale

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# Stability of toroidal magnetic field

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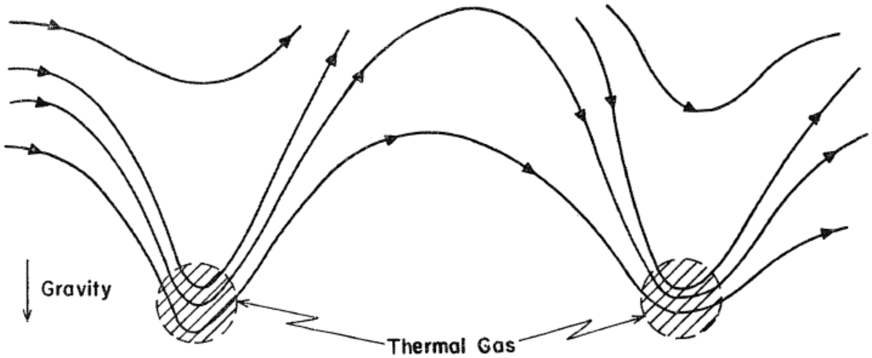
$$\frac{g_z}{c_s^2}\partial_z \ln\left(B_{(\varphi)}\varpi\right) + \frac{m^2}{\varpi^2} < 0$$

Mainly focusing on $\varpi \gg \varpi_c$




Growth time scale $\sim$ Alfvén time scale $R/v_A$

**What happens after the onset of the instability ?**

# Axisymmetric General Relativistic MHD

**Prediction from initial condition**

Note : initial condition = equilibrium configuration (Recall $B_{(\varphi)} \propto \rho^k \varpi^{2k-1}$, $\left(\frac{1}{\varpi} - \frac{1}{\varpi_c}\right) \partial_\varpi \ln \left(\frac{B_{(\varphi)}}{\rho\varpi}\right) > 0$)

- k=1 $\Rightarrow$ (Marginally) Stable
- $k \geq 2 \Rightarrow$ Unstable

Instability = Interchange instability

**Exploring this instability by Numerical Relativity**

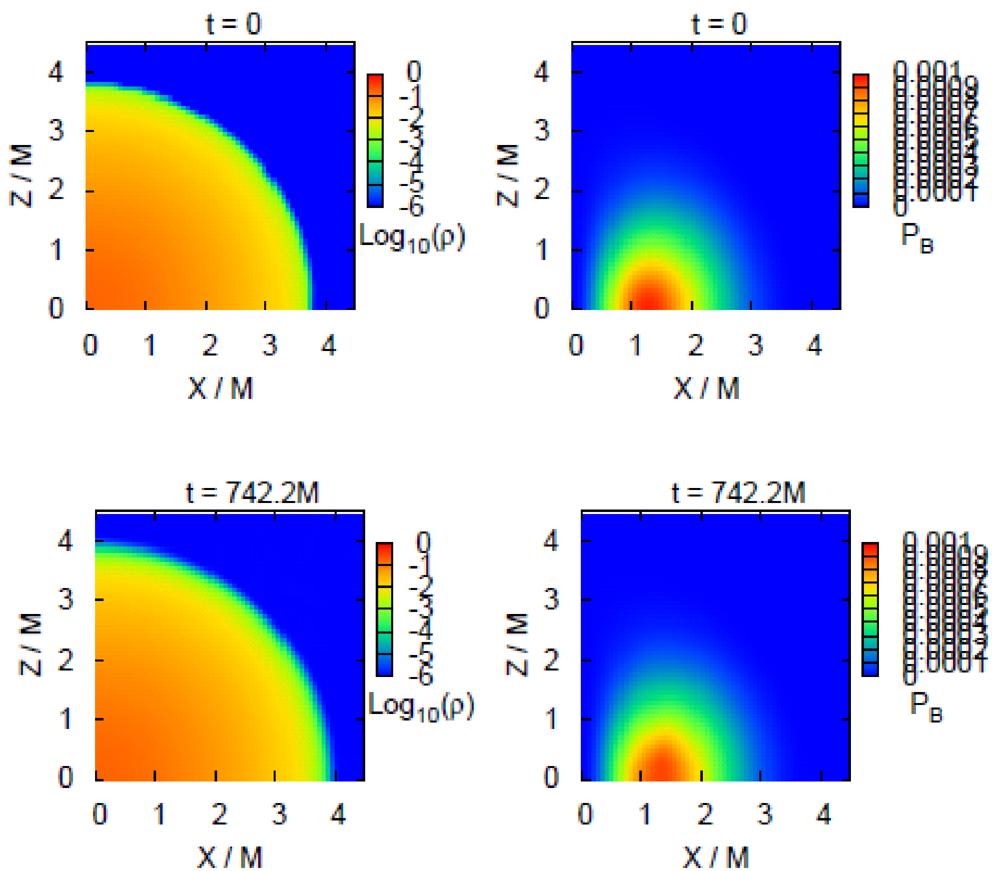
# Axisymmetric General Relativistic MHD

## k=1 configuration(Stable)

Density | Magnetic field energy density

t=0

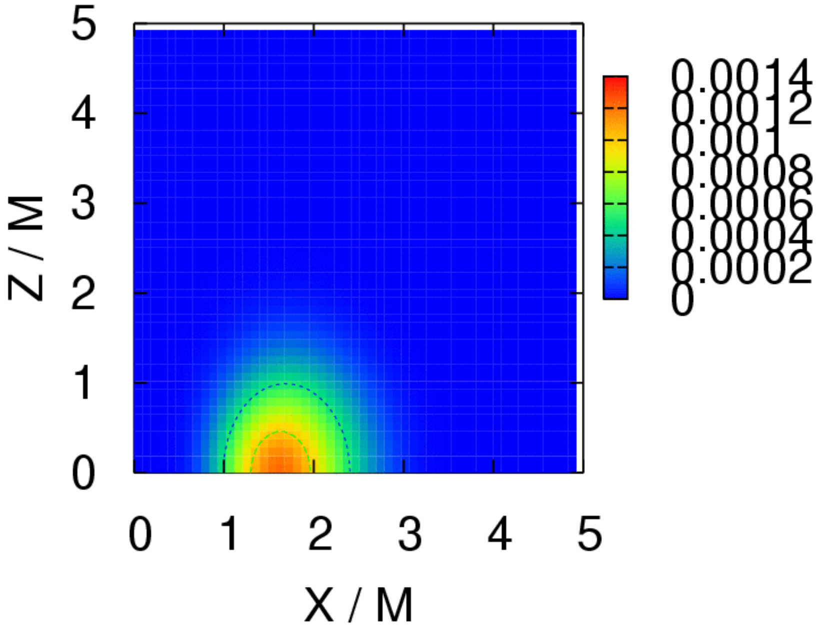
$t \approx 11 t_A$



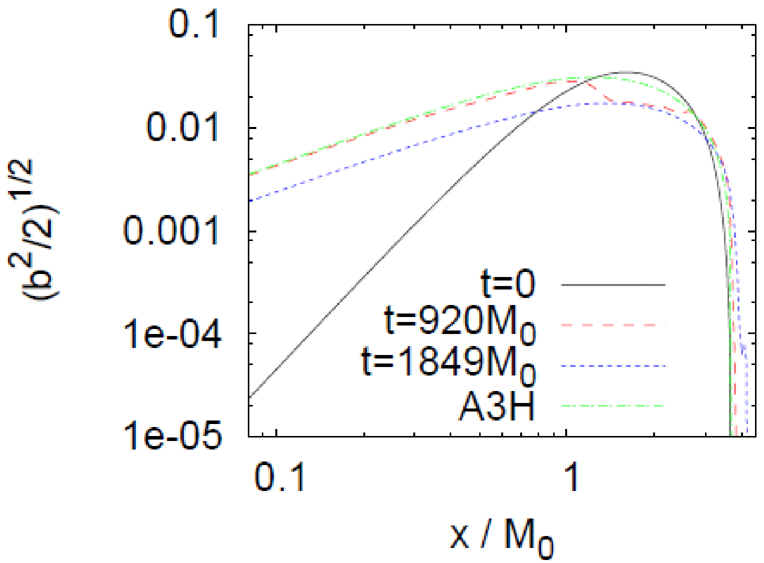
# Axisymmetric General Relativistic MHD

## k=2 configuration(Unstable)

Magnetic field energy density



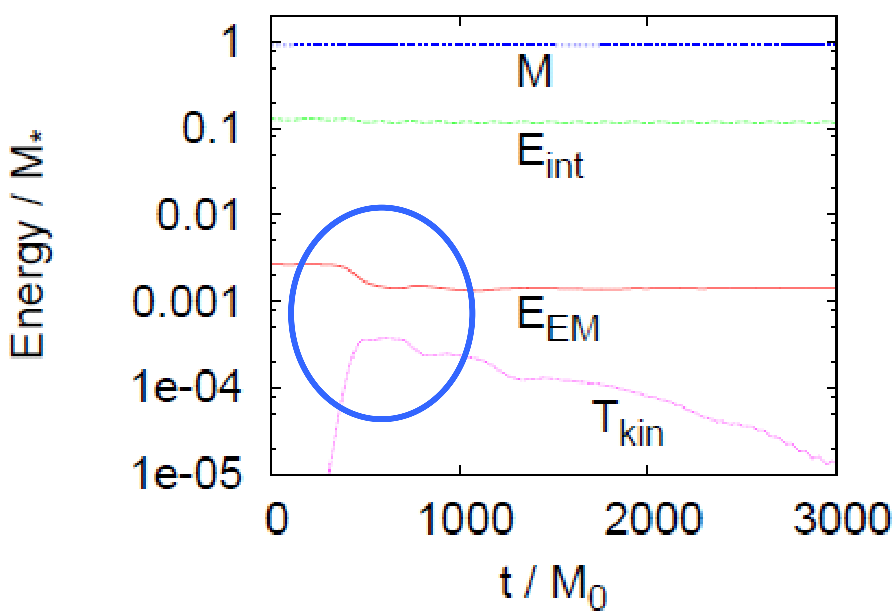

Snapshot

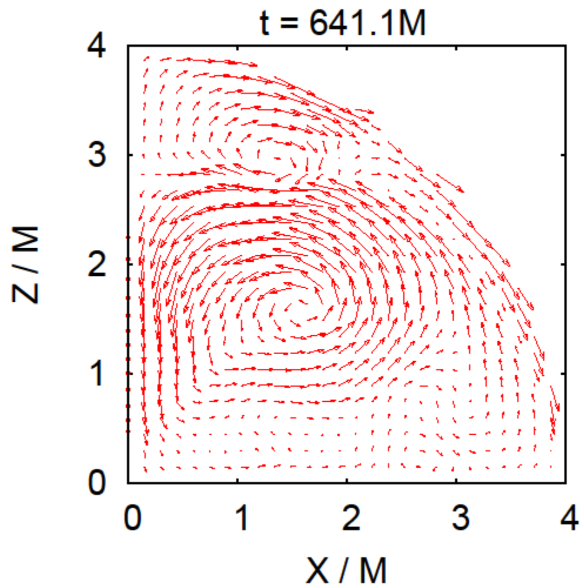



**Unstable configuration (k=2) comes to the stable one (k=1)**

# Axisymmetric General Relativistic MHD

Magnetic energy $\Rightarrow$ Kinetic energy

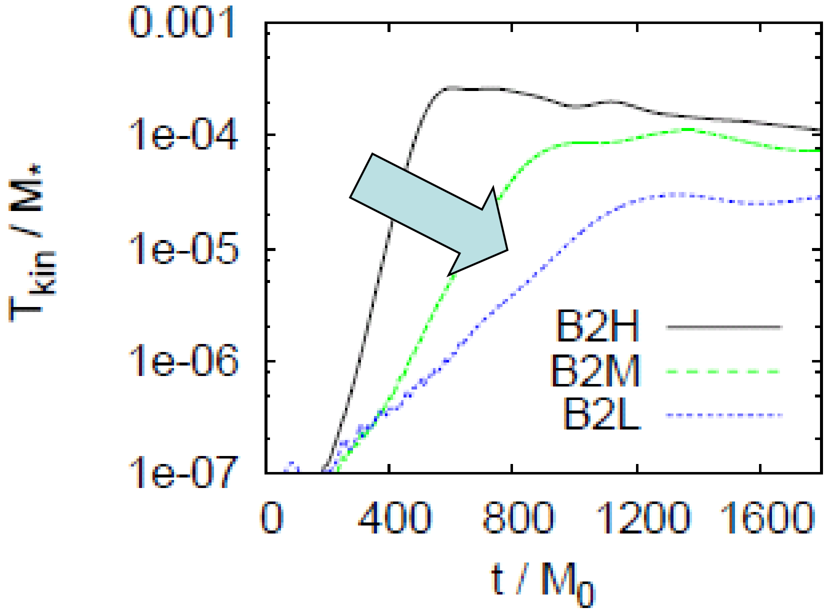



Interchange instability induces the circular motion on the meridional plane and settles down to a quasi equilibrium state.

# Axisymmetric General Relativistic MHD

Magnetic field ↘




Time scale ≈ Alfvén time scale

Time scale almost agrees with linear perturbation analysis

$T_{\text{kin}} \approx 0.1 E_{\text{EM}}$

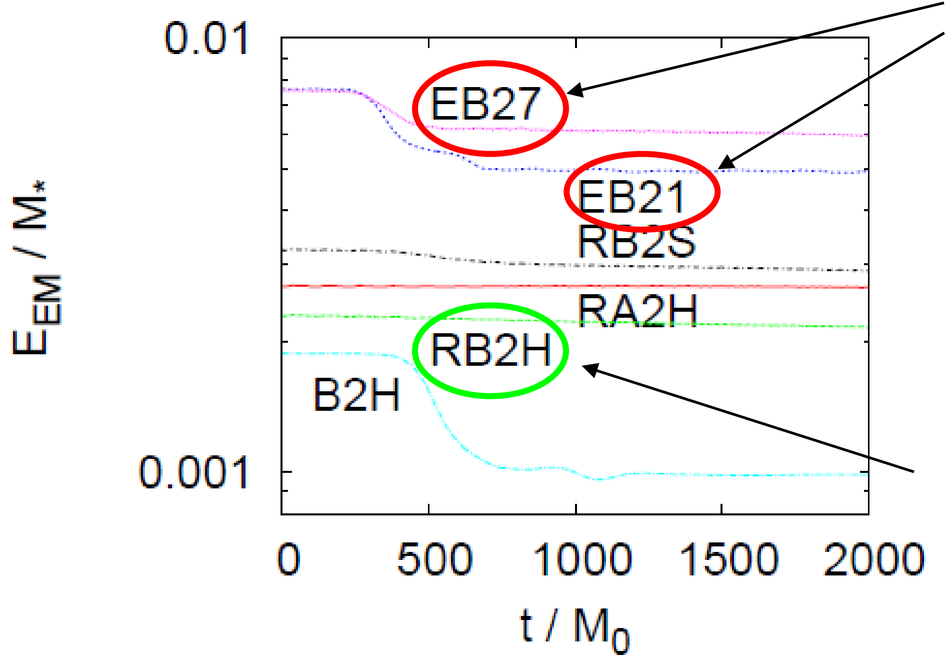
# Axisymmetric General Relativistic MHD

+Rigid Rotation

Instability criterion

$$4\left(1+\frac{v_A^2}{c_s^2}\right)\Omega^2 + v_A^2\left(\frac{1}{\varpi}-\frac{1}{\varpi_c}\right)\partial_\varpi \ln\left(\frac{B_{(\varphi)}}{\rho\varpi}\right) > 0$$

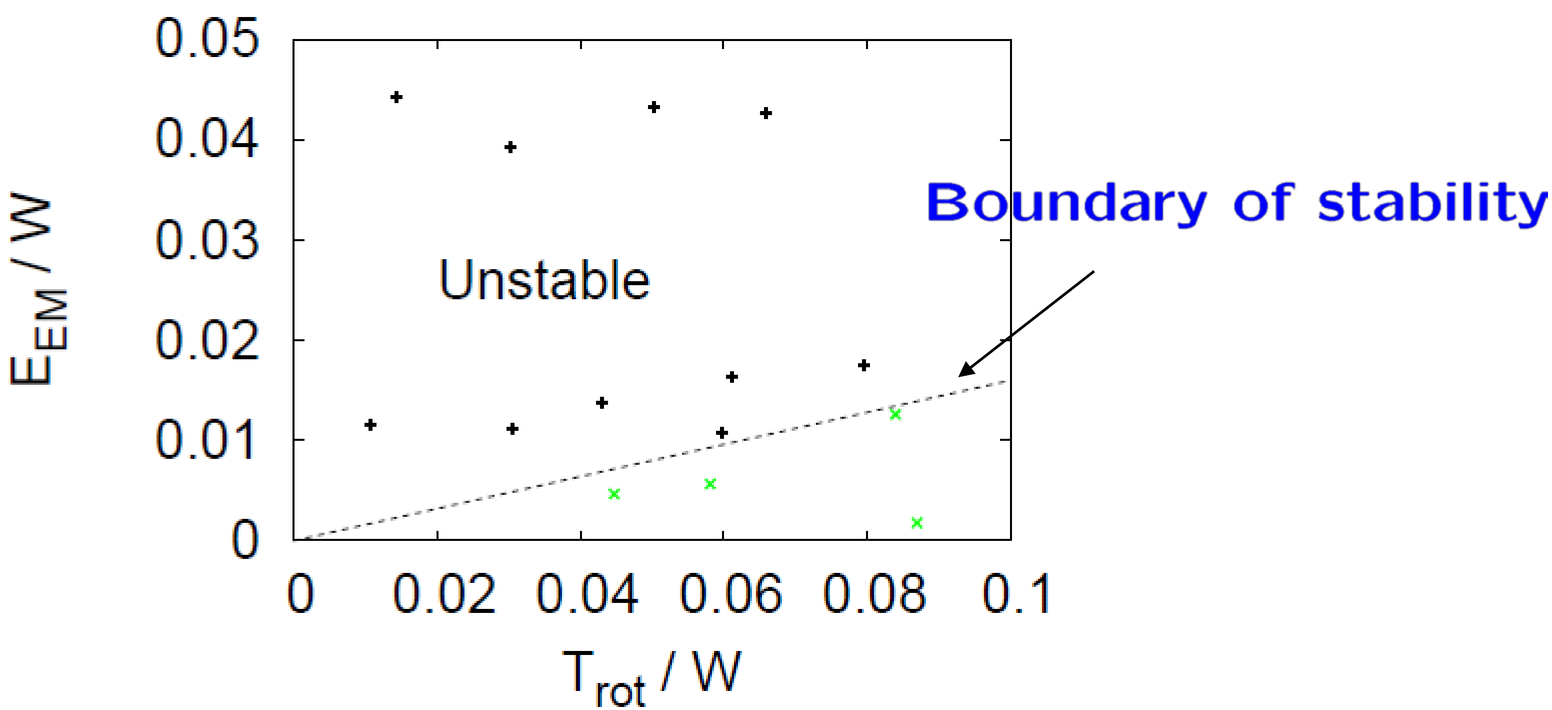
$E_{\rm EM}/E_{\rm rot} \approx 0.7 - 0.8$




$E_{\rm EM}/E_{\rm rot} \approx 0.1$

# Axisymmetric General Relativistic MHD

## k=2 configuration




If the instability ignites,

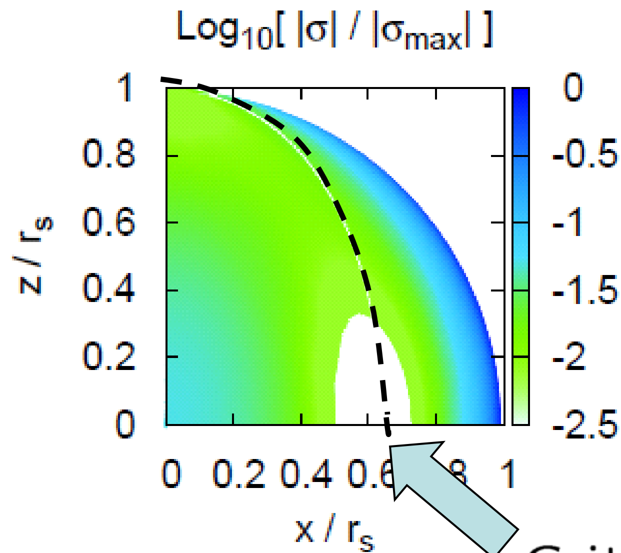
$T_{\rm kin} \sim 10^{50} \left(\frac{B_{(\varphi)}}{10^{16}\rm G}\right)^2 \left(\frac{R}{15\rm km}\right)^3 \rm erg \sim$ 10% of the energy for SN

# 3D General Relativistic MHD

Prediction from initial condition

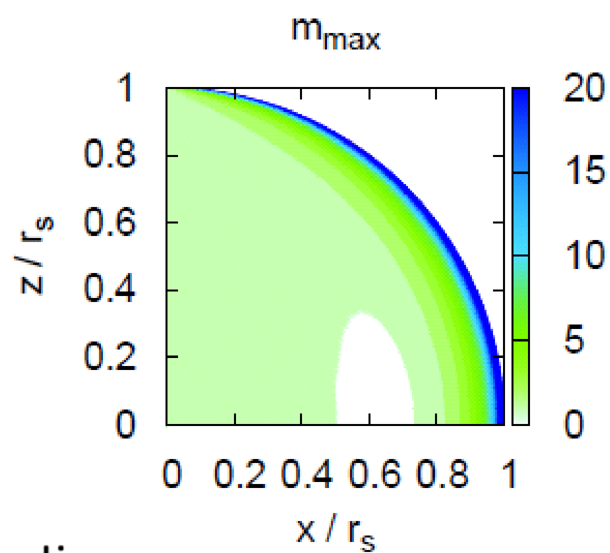
**k=1 configuration** (stable against m=0 mode)

Growth rate




Critical radius

Most unstable mode ($e^{im\varphi}$)




Primarily instability = Parker instability

# 3D General Relativistic MHD

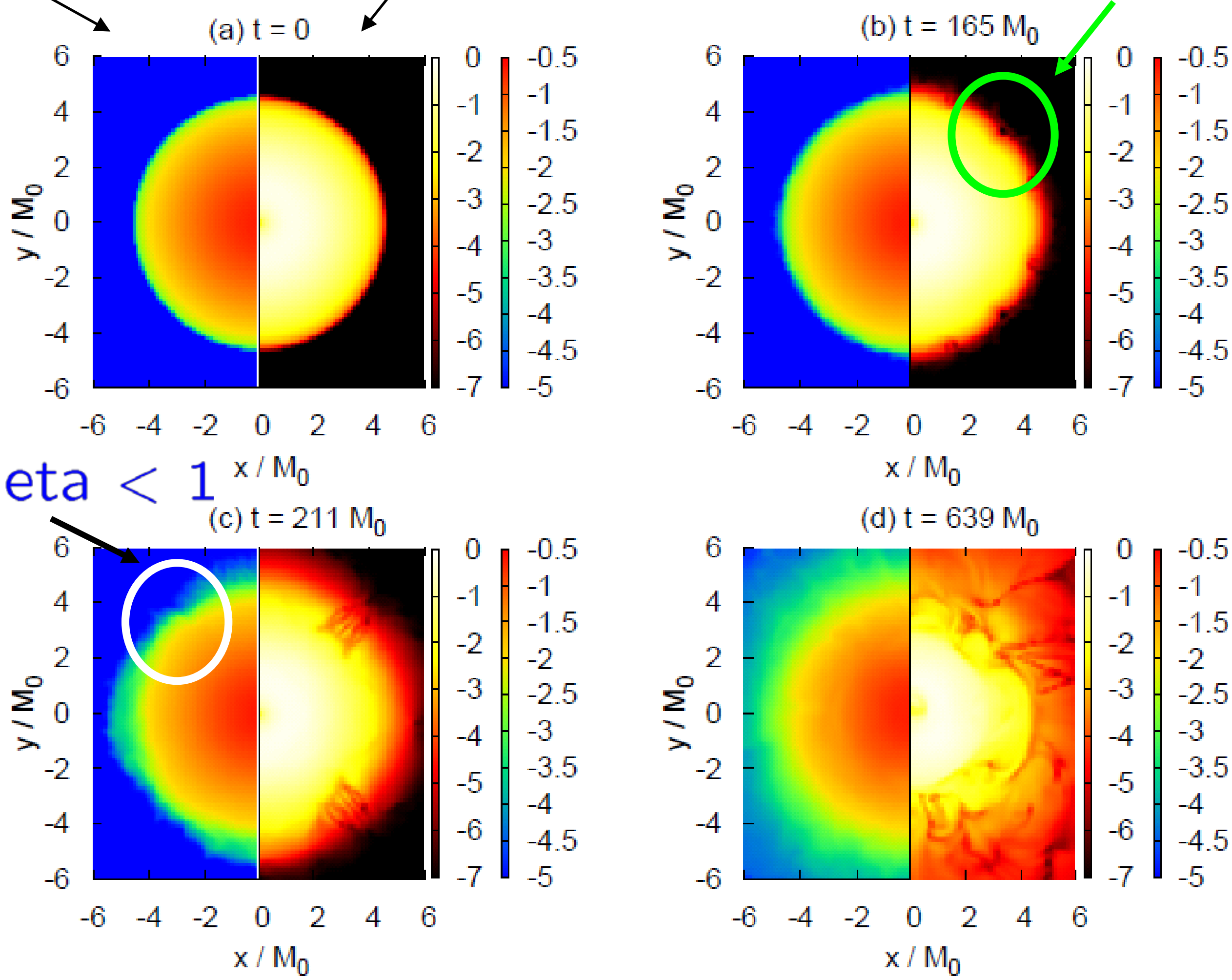
Animation

Density

Magnetic energy density

Parker instability

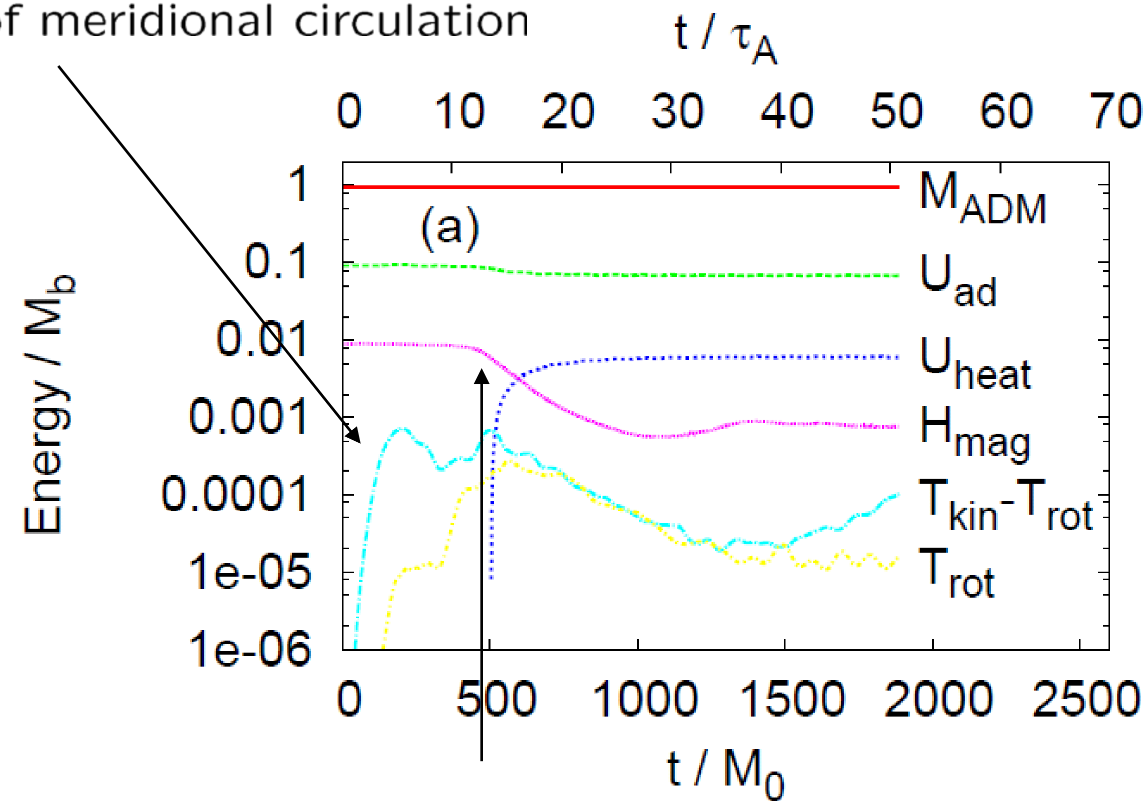
Plasma beta < 1




Turbulent magnetic field

# 3D General Relativistic MHD

Excitation of meridional circulation




Turbulent magnetic field & shock heating

**Magnetic field never reaches equilibrium state**

# 3D General Relativistic MHD

## +Rigid Rotation

Instability criterion

$$-\Omega^2\varpi\partial_\varpi(\ln\Omega^2) + v_A^2\left(\frac{2}{\varpi} - \frac{2}{\varpi_c}\right)\partial_\varpi \ln\left(\frac{B_{(\varphi)}}{\rho\varpi}\right) > \frac{m^2 v_A^2}{\varpi^2}$$

Note that $|\Omega\varpi| \gg |v_A|$

Recall that for k=1 $\Rightarrow B_{(\varphi)} \propto \rho\varpi$

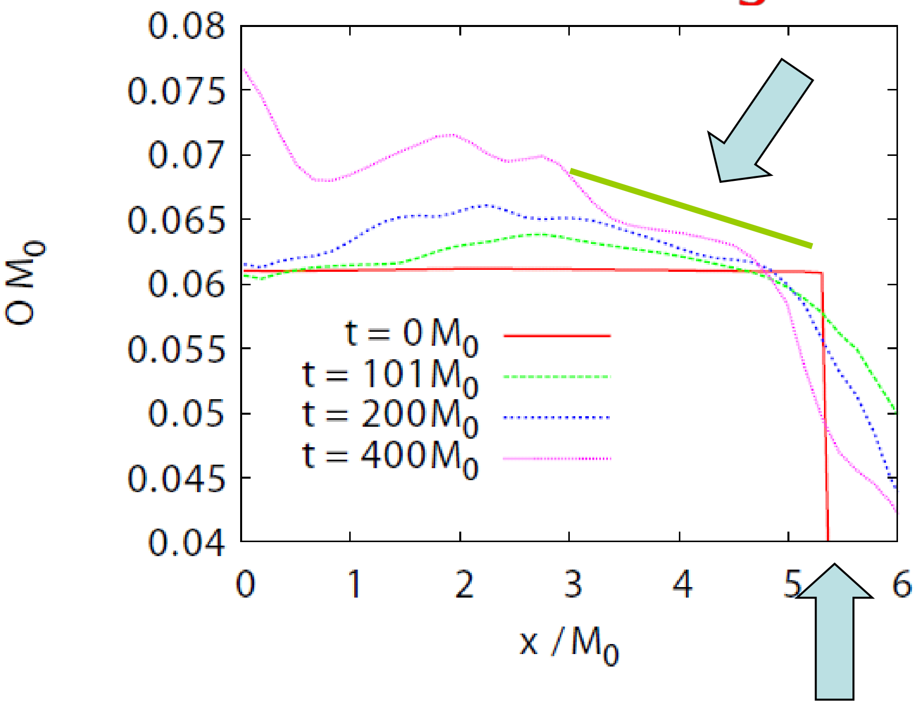
Rapid & rigid rotation stabilizes the instability

Animation

# 3D General Relativistic MHD

Angular velocity profile

Negative gradient




Stellar surface

Instability criterion

$-\Omega^2 \varpi \partial_\varpi (\ln \Omega^2)$

$+ v_A^2 \left( \frac{2}{\varpi} - \frac{2}{\varpi_c} \right) \partial_\varpi \ln \left( \frac{B_{(\varphi)}}{\rho \varpi} \right)$

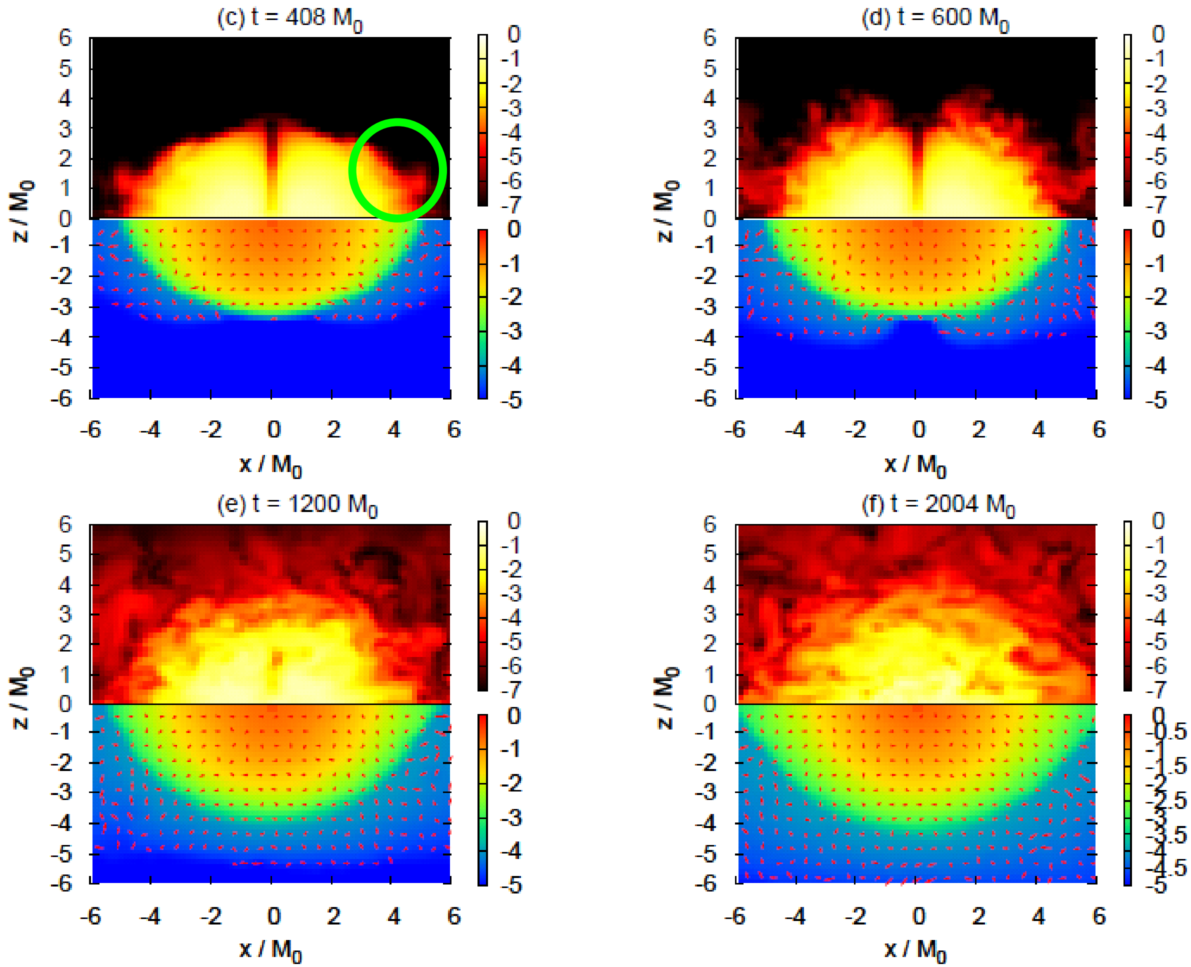
$> \frac{m^2 v_A^2}{\varpi^2}$

Note that $|\Omega \varpi| \gg |v_A|$

# 3D General Relativistic MHD

Mag. ene. density

Density




**Magnetic field never reaches equilibrium state even if fast rotation**

## Summary

- Toroidally magnetized relativistic star in equilibirium
- Interhcange instability with 2D GRMHD
- Parker and/or Tayler instability with 3D GRMHD

## Future issure

- Startification by composition gradient (Stabilizing agent)
- Spruit dynamo (Tayler instability and magnetic winding)