原子核物質の一次相転移における混合相と状態方程式

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- Non uniform structures at first-order phase transitions.
- EOS of matter (mixed phase).

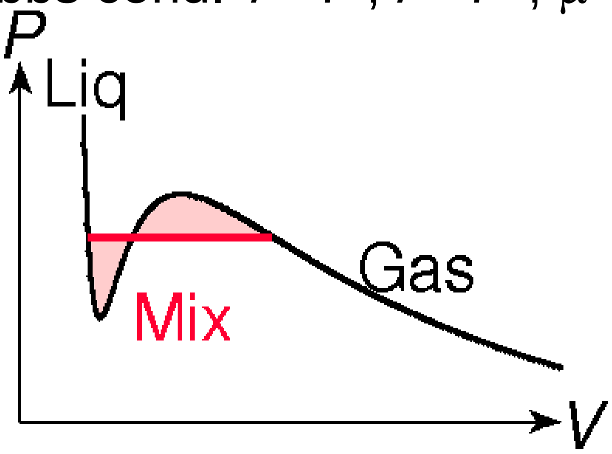
Phase transitions in nuclear matter

Liquid-gas, neutron drip, meson condensation,
hyperon mixture, quark deconfinement,
color super-conductivity, etc.

EOS of mixed phase in first order phase transition

- Single component (e.g. water)

Maxwell construction **satisfies** the Gibbs cond. $T^{I}=T^{II}$, $P^{I}=P^{II}$, $\mu^{I}=\mu^{II}$.



- Many components (e.g. water+ethanol)

Gibbs cond. $T^{I}=T^{II}$, $P_i^{I}=P_i^{II}$, $\mu_i^{I}=\mu_i^{II}$.
No Maxwell construction !

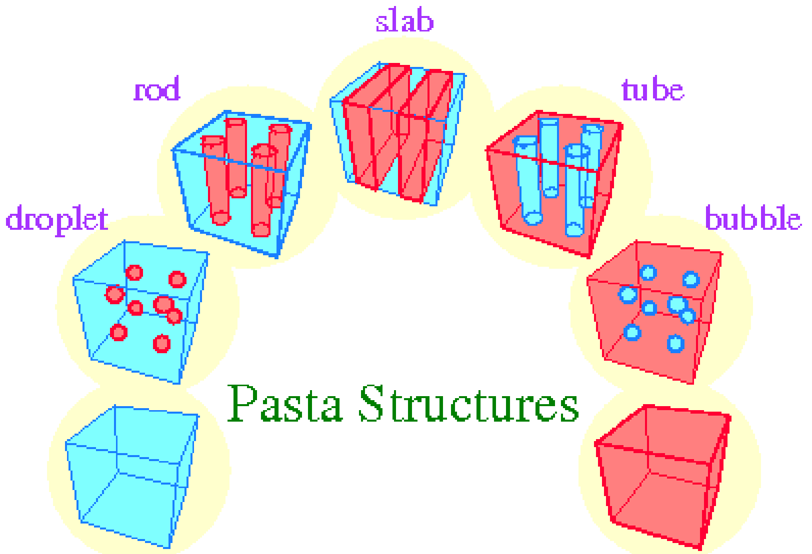
- Many charged components (nuclear matter)

Gibbs cond. $T^{I}=T^{II}$, $\mu_i^{I}=\mu_i^{II}$.
No Maxwell construction !
No constant *pressure* !

$$\frac{dP_i}{dr} = -\frac{\partial U_i(\rho_i;r)}{\partial r}$$

In the mixed phase with charged particles, non-uniform "Pasta" structures are expected.

[Ravenhall *et al*, PRL 50 (1983) 2066]



Depending on the density, geometrical structure of mixed phase changes from droplet, rod, slab, tube and to bubble configuration.

Quite **different** from a **bulk picture** of mixed phase.
We have to take into account the <u>effect of the structure</u> when we calculate the EOS.

Surface tension & Coulomb

Low density nuclear matter

- Collapsing stage of supernova explosion

  → Liquid-gas phase transition

  At T=0, (1) electron phase (2) nuclear phase

  At T>0, (1) dilute phase (2) dense phase

- Neutron star crust

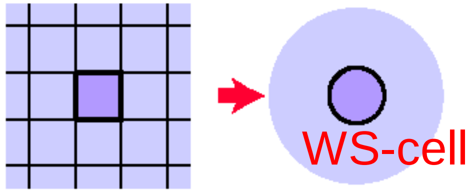
  → Neutron drip

  At T=0, (1) neutron phase (2) nuclear phase

Numerical calculation of mixed-phase

• Assume regularity in structure: divide whole space into equivalent and neutral cells with a geometrical symmetry (3D: sphere, 2D : cylinder, 1D: plate).

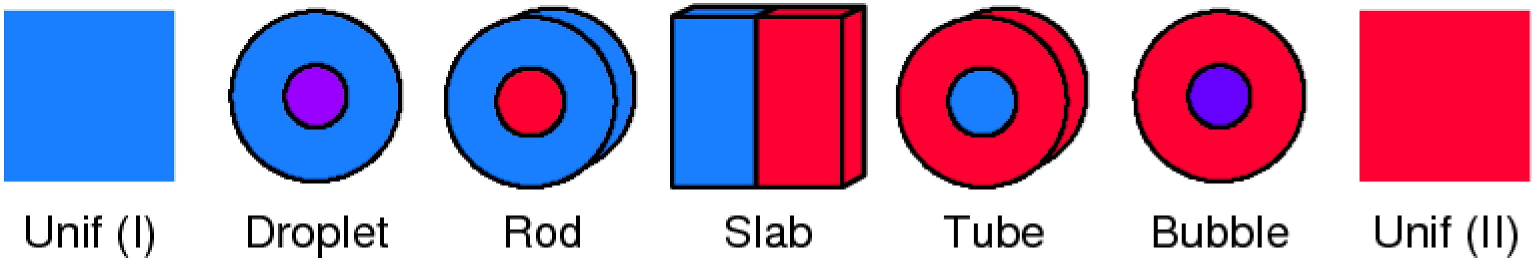
→ Wigner-Seitz cell approx.



• Give a geometry (Unif/Dropl/Rod/...) and a baryon density ρ_B.

• Solve the field equations numerically. Optimize the cell size (choose the energy-minimum).

• Choose an energy-minimum geometry among 7 cases (Unif (I), droplet, rod, slab, tube, bubble, Unif (II)).



Field equations to be solved

Relativistic Mean Field (RMF) model:

Lorentz-covariant thermodynamic potential Ω with baryon densities, meson fields (σ, ω, ρ), electron density and the Coulomb potential is determined.

Local density approx:

Local density approximation for baryons and electron →Thomas Fermi

Consistent treatment for potentials and densities:

→ Coulomb screening by charged particles

[T.M. et al,PRC72(2005)015802; Rec.Res.Dev.Phys,7(2006)1]

$$\Omega = \Omega_B + \Omega_M + \Omega_e$$

(T=0)

$$\Omega_B = \sum_{a=p,n} \int d^3r \int_0^{p^F{}_a} \frac{d^3p}{(2\pi\hbar)^3} \sqrt{m^{*2} + p^2} - \rho_a(\mu_a - U_a), \quad \Omega_e = \int d^3r \left[\frac{1}{8\pi e^2} (\nabla V_{\mathrm{Coul}})^2 - \frac{(\mu_e - V_{\mathrm{Coul}})^4}{12\pi^2} \right]$$

$$\Omega_M = \int d^3r \left[\frac{(\nabla\sigma)^2 + m_\sigma^2\sigma^2}{2} + U_\sigma - \frac{(\nabla\omega)^2 + m_\omega^2\omega^2}{2} - \frac{(\nabla R)^2 + m_\rho^2 R^2}{2} \right]$$

(Non local effects for mesons)

$$U_p = g_{\omega N}\omega_0(\mathbf{r}) + g_{\rho N}R_0(\mathbf{r}) - V_{\mathrm{Coul}}(\mathbf{r}), \quad U_n = g_{\omega N}\omega_0(\mathbf{r}) - g_{\rho N}R_0(\mathbf{r})$$

$$m_N^*(\mathbf{r}) = m_N - g_{\sigma N}\sigma(\mathbf{r}), \qquad \rho_a^s(\mathbf{r}) = \int_0^{k_{F,a}(\mathbf{r})} \frac{d^3k}{4\pi} \frac{m_N^*(\mathbf{r})}{\sqrt{m_N^*(\mathbf{r})^2 + k^2}}$$

$$\nabla^2 V_{\mathrm{Coul}}(\mathbf{r}) = 4\pi e^2 \rho_{\mathrm{ch}}(\mathbf{r}), \qquad \rho_{\mathrm{ch}}(\mathbf{r}) = \rho_p(\mathbf{r}) - \rho_e(\mathbf{r})$$

EOM for fields

Chemical equilibrium fully consistent with all the density distributions and fields.

$$\nabla^2 \sigma(\mathbf{r}) = m_\sigma^2 \sigma(\mathbf{r}) + \frac{dU}{d\sigma} - g_{\sigma N}\left(\rho_n^s(\mathbf{r}) + \rho_p^s(\mathbf{r})\right),$$

$$\nabla^2 \omega_0(\mathbf{r}) = m_\omega^2 \omega_0(\mathbf{r}) - g_{\omega N}\left(\rho_n(\mathbf{r}) + \rho_p(\mathbf{r})\right),$$

$$\nabla^2 R_0(\mathbf{r}) = m_\rho^2 R_0(\mathbf{r}) - g_{\rho N}\left(\rho_n(\mathbf{r}) - \rho_p(\mathbf{r})\right),$$

$$\nabla^2 V_{\mathrm{Coul}}(\mathbf{r}) = 4\pi e^2 \rho_{\mathrm{ch}}(\mathbf{r}), \qquad \rho_{\mathrm{ch}}(\mathbf{r}) = \rho_p(\mathbf{r}) - \rho_e(\mathbf{r}),$$

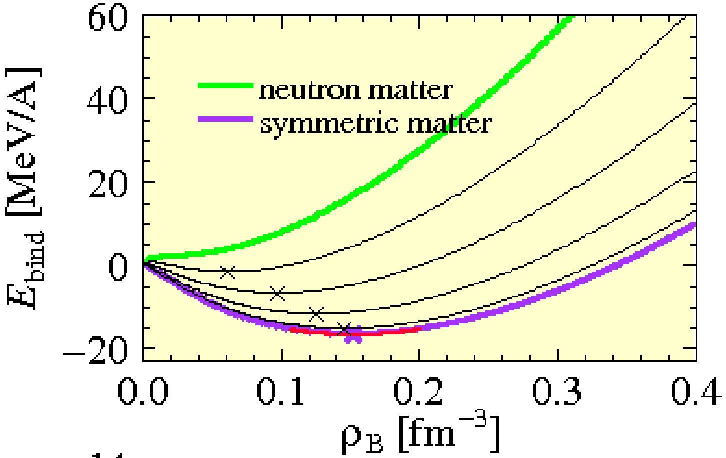
$$\mu_n = \mu_B = \mu_p + \mu_e, \qquad \mu_e = \left(3\pi\rho_e(\mathbf{r})\right)^{1/3} + V_{\mathrm{Coul}}(\mathbf{r}),$$

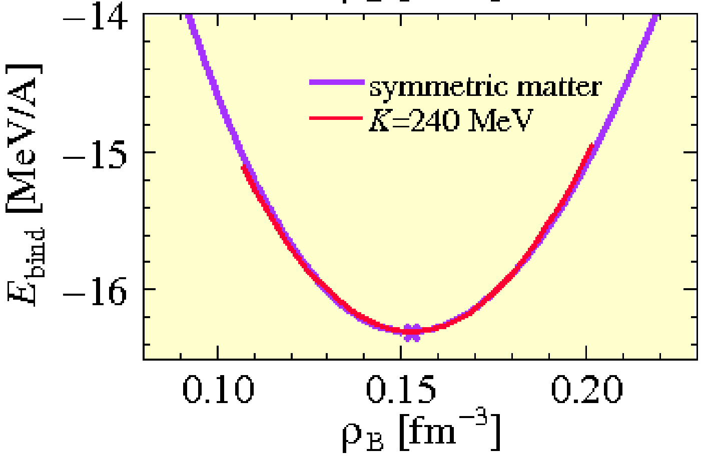
$$\mu_n = \sqrt{k_{F,n}(\mathbf{r})^2 + m_N^*(\mathbf{r})^2} + g_{\omega N}\omega_0(\mathbf{r}) - g_{\rho N} R_0(\mathbf{r}),$$

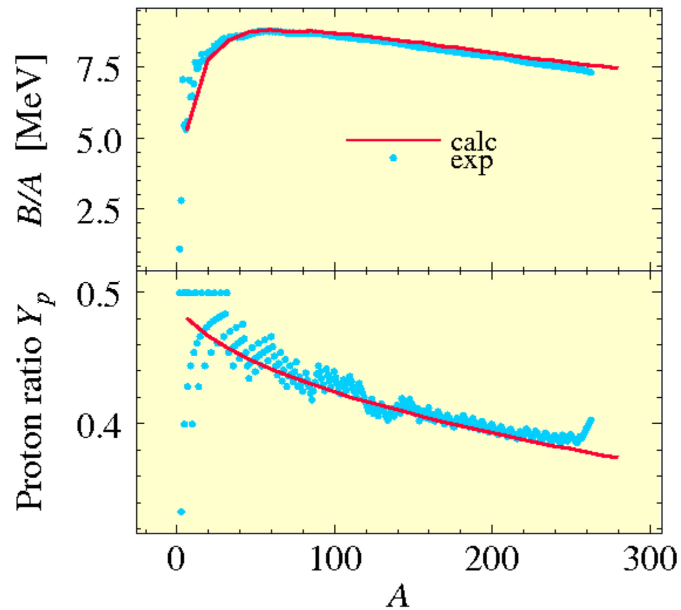
$$\mu_p = \sqrt{k_{F,p}(\mathbf{r})^2 + m_N^*(\mathbf{r})^2} + g_{\omega N}\omega_0(\mathbf{r}) + g_{\rho N} R_0(\mathbf{r}) - V_{\mathrm{Coul}}(\mathbf{r}),$$

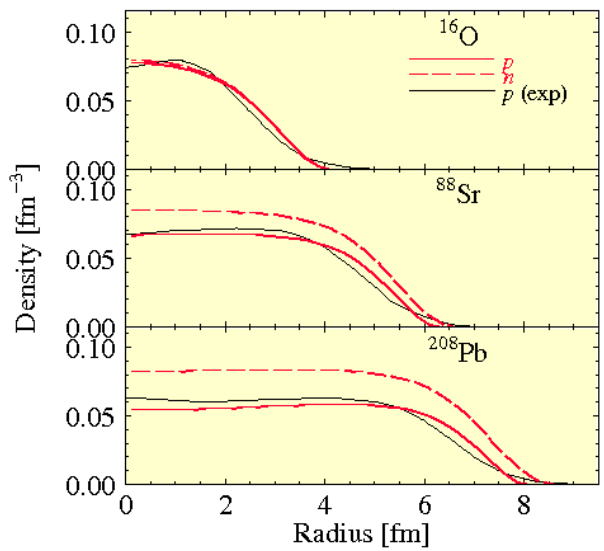
$$m_N^*(\mathbf{r}) = m_N - g_{\sigma N}\sigma(\mathbf{r}), \qquad \rho_a^s(\mathbf{r}) = 2\int_0^{p_{F,a}(\mathbf{r})} \frac{d^3p}{(2\pi\hbar)^3} \frac{m_N^*(\mathbf{r})}{\sqrt{m_N^*(\mathbf{r})^2 + p^2}}$$

Properties of uniform matter and nuclei



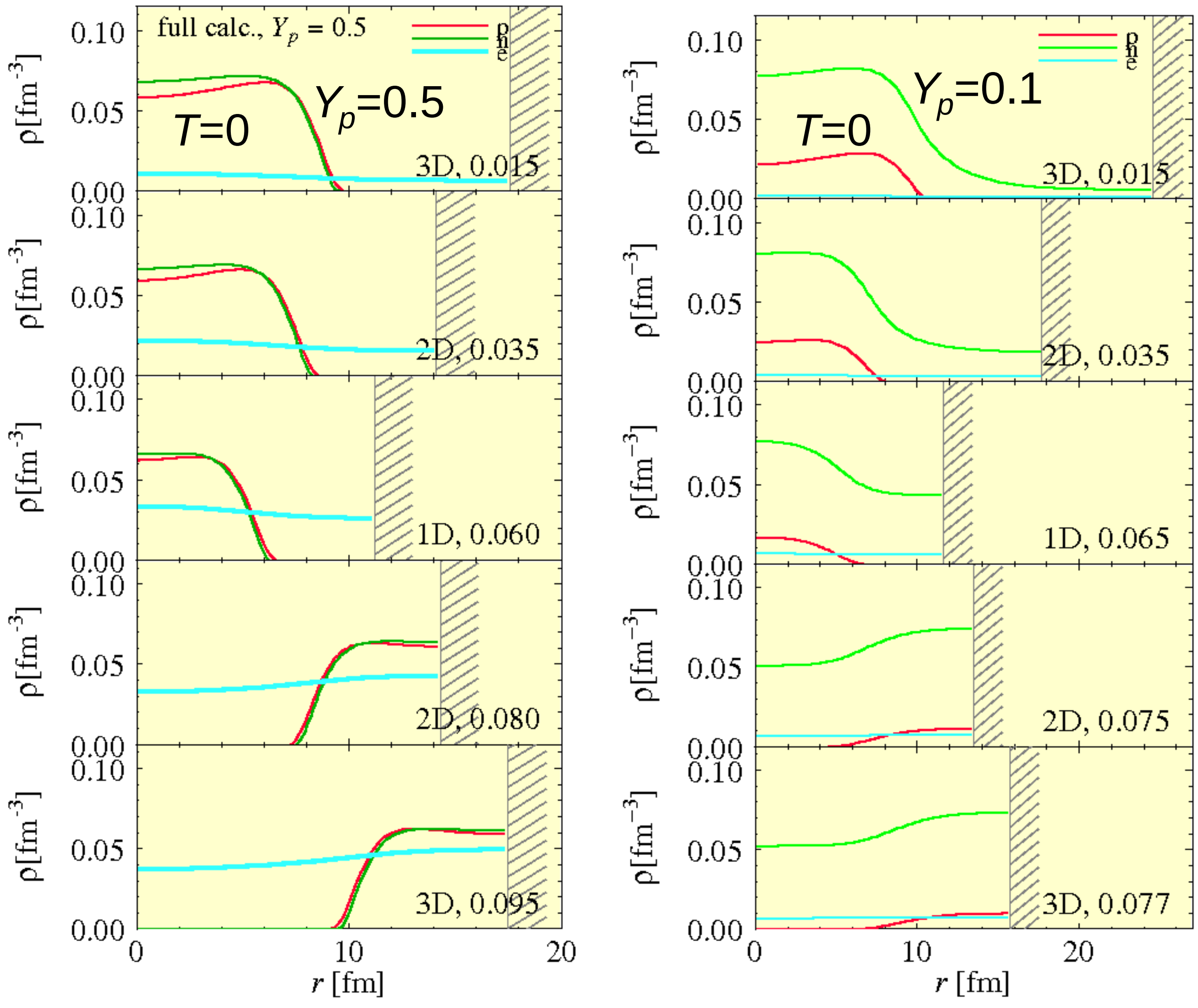




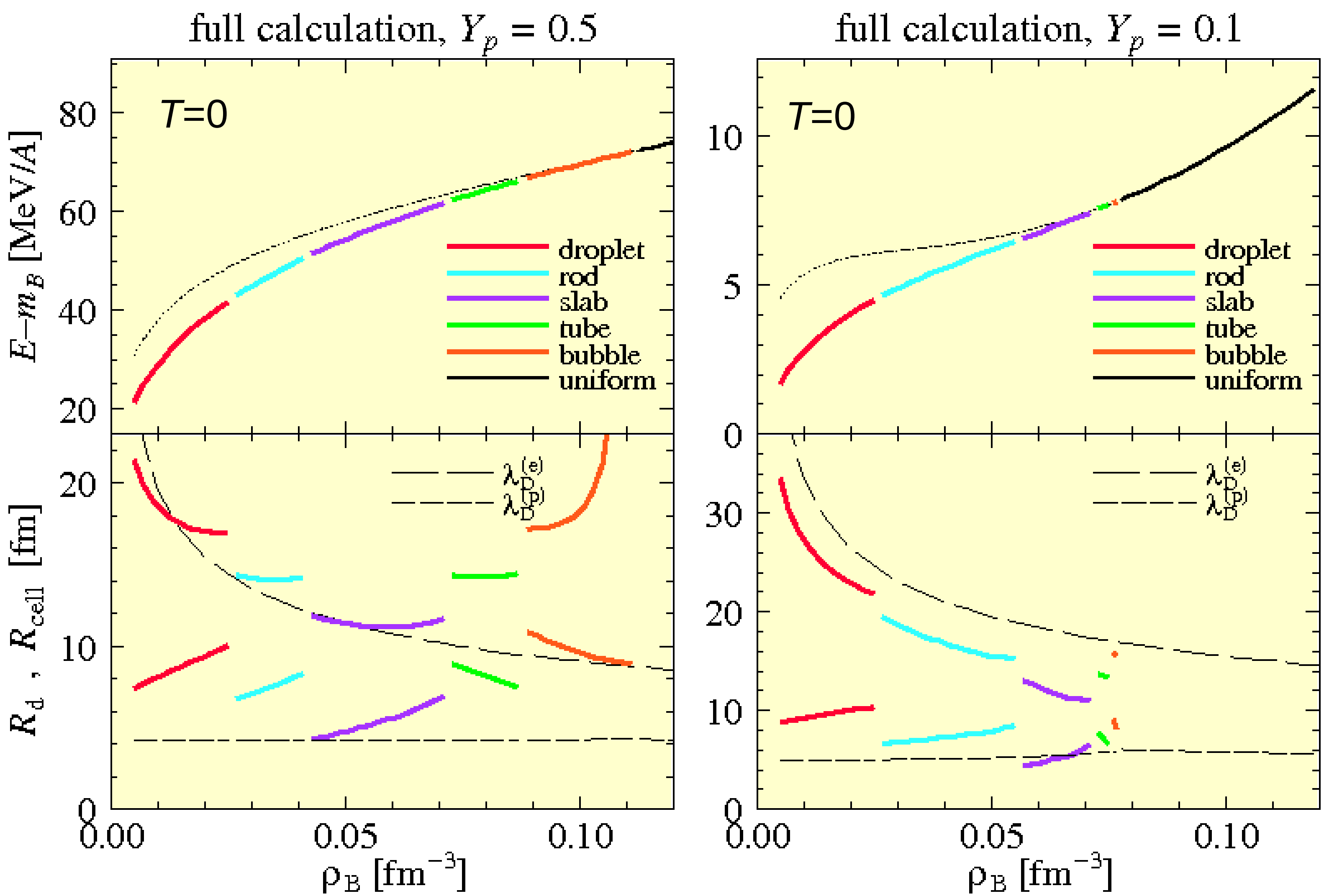


Pasta structures in matter (case of fixed Y_p)

Density profiles in WS cells



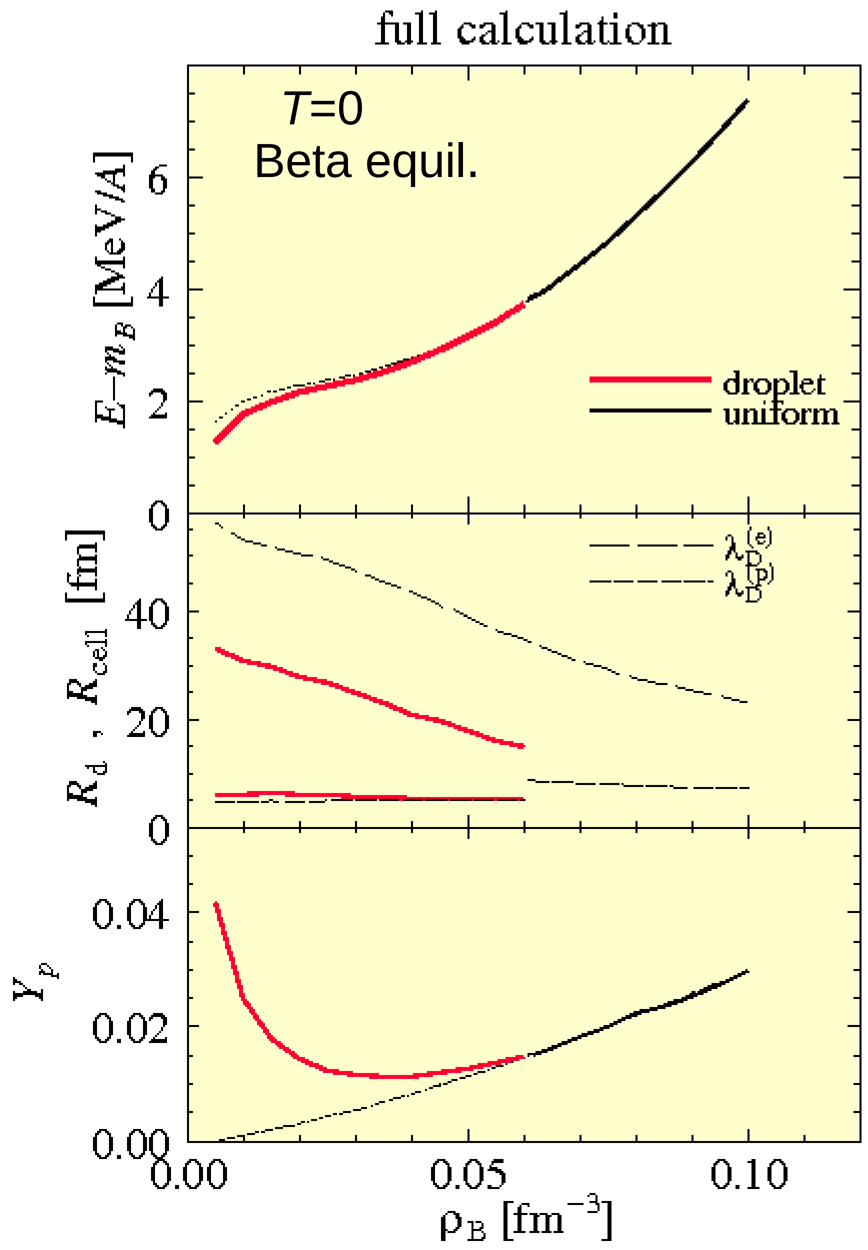
EOS and the structure size



Beta-equilibrium case

Only "droplet" structure appears.

The change of EOS due to the non-uniform structure is small.



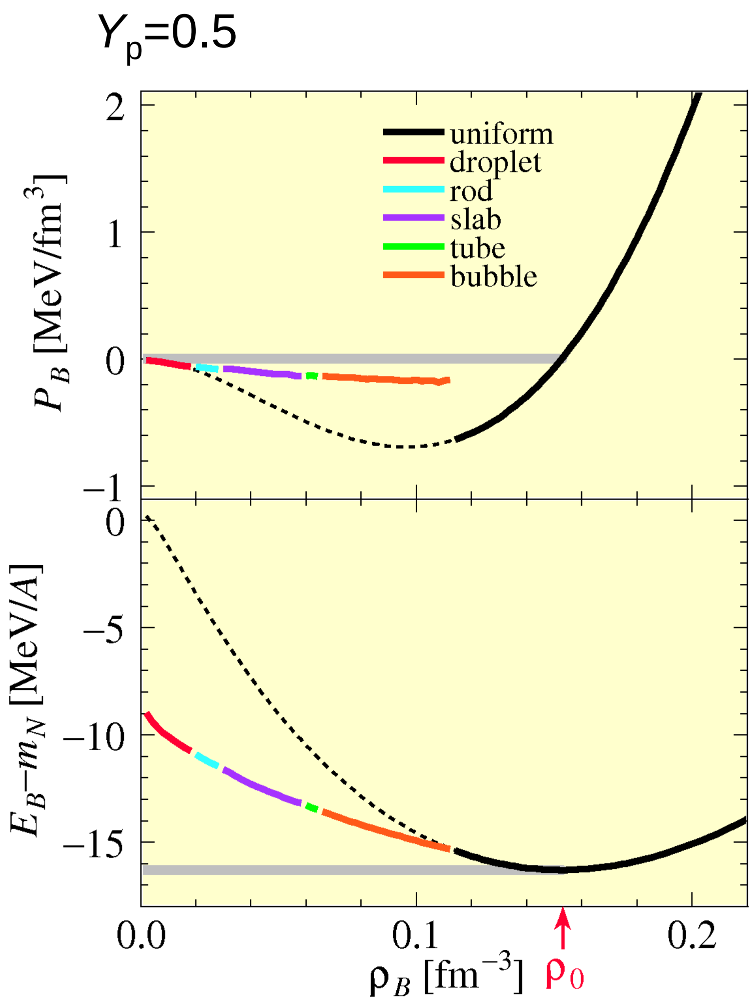
Clustering mechanism

Total pressure is positive due to electron pressure.

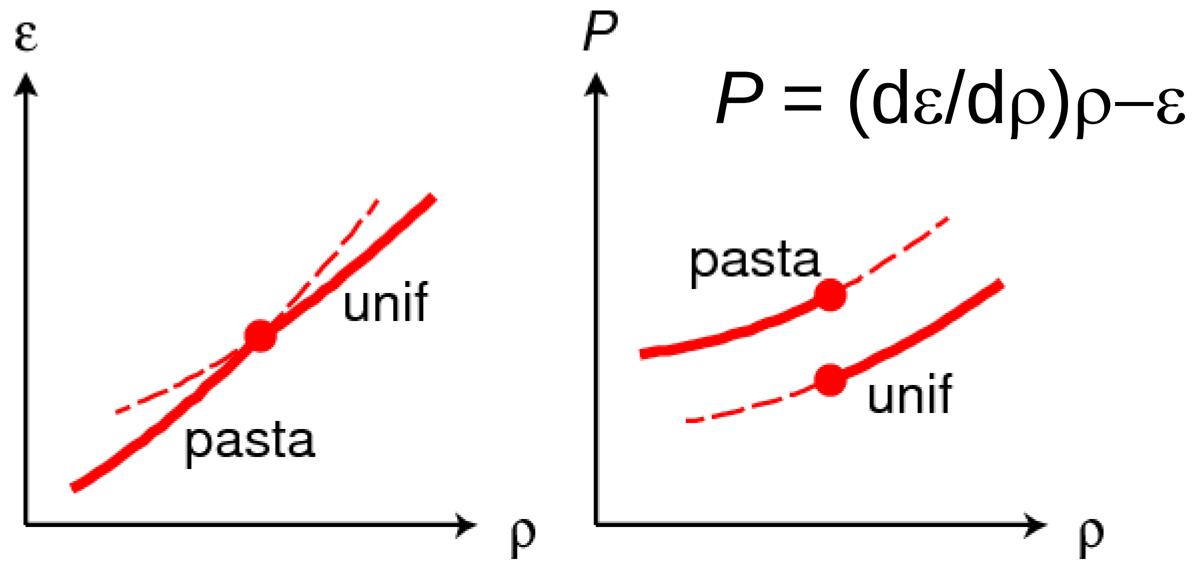
Negative baryon pressure at $\rho_B < \rho_0$
→ unstable
→ clustering

But in a density region near ρ_0, uniform matter is favored due to the surface and the Coulomb energies of pasta structures.

Notice : Jump of pressure.
→ is it OK?



Jump of pressure may occur



Energy density should be continuous.
But the pressure can have a jump.

→ → Maxwell construction between pasta and uniform phases ?

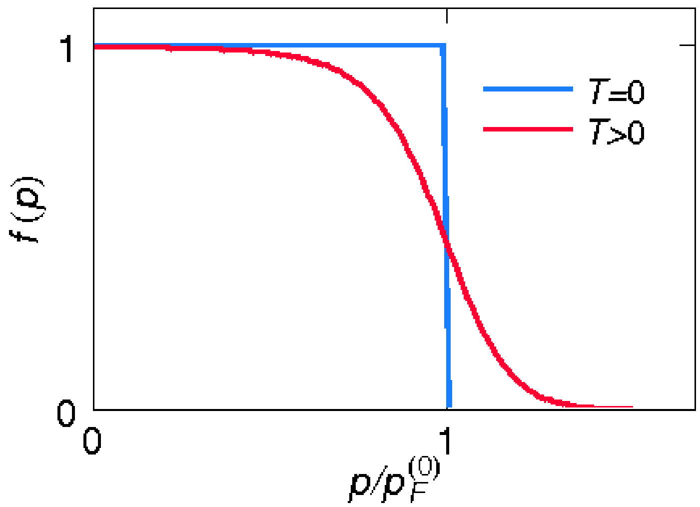
Finite temperature matter

Extension to finite temperature

Baryon and electron :

Employ the Fermi-Dirac distribution for the momentum distribution $f(p)$

→ T dependence of kinetic energy, chemical potential, and baryon scalar density.



Geometry and size of the cell :

Choose minimum free energy F/A for given baryon density.

EOM for fields

in the case of T=0

$$\nabla^2\sigma(\mathbf{r}) = m_\sigma^2\sigma(\mathbf{r}) + \frac{dU}{d\sigma} - g_{\sigma N}\left(\rho_n^s(\mathbf{r}) + \rho_p^s(\mathbf{r})\right),$$

$$\nabla^2\omega_0(\mathbf{r}) = m_\omega^2\omega_0(\mathbf{r}) - g_{\omega N}\left(\rho_n(\mathbf{r}) + \rho_p(\mathbf{r})\right),$$

$$\nabla^2 R_0(\mathbf{r}) = m_\rho^2 R_0(\mathbf{r}) - g_{\rho N}\left(\rho_n(\mathbf{r}) - \rho_p(\mathbf{r})\right),$$

$$\nabla^2 V_{\mathrm{Coul}}(\mathbf{r}) = 4\pi e^2\rho_{\mathrm{ch}}(\mathbf{r}), \qquad \rho_{\mathrm{ch}}(\mathbf{r}) = \rho_p(\mathbf{r}) - \rho_e(\mathbf{r}),$$

$$\mu_n = \mu_B = \mu_p + \mu_e, \qquad \mu_e = \left(3\pi\rho_e(\mathbf{r})\right)^{1/3} + V_{\mathrm{Coul}}(\mathbf{r}),$$

$$\mu_n = \sqrt{k_{F,n}(\mathbf{r})^2 + m_N^*(\mathbf{r})^2} + g_{\omega N}\omega_0(\mathbf{r}) - g_{\rho N}R_0(\mathbf{r}),$$

$$\mu_p = \sqrt{k_{F,p}(\mathbf{r})^2 + m_N^*(\mathbf{r})^2} + g_{\omega N}\omega_0(\mathbf{r}) + g_{\rho N}R_0(\mathbf{r}) - V_{\mathrm{Coul}}(\mathbf{r}),$$

$$m_N^*(\mathbf{r}) = m_N - g_{\sigma N}\sigma(\mathbf{r}), \qquad \rho_a^s(\mathbf{r}) = 2\int_0^{p_{F,a}(\mathbf{r})} \frac{d^3p}{(2\pi\hbar)^3}\frac{m_N^*(\mathbf{r})}{\sqrt{m_N^*(\mathbf{r})^2 + p^2}}$$

T=0

$$\rho_{T=0} = \frac{1}{3\pi^2}(\mu - m)^3$$

$$\rho^s_{T=0} = 2\int_0^{p_F} \frac{d^3p}{(2\pi\hbar)^3} \frac{m_N^*}{\sqrt{m_N^{*\,2} + k^2}}$$

$$f(p) = \theta(p_F - p)$$

T>0

$$\rho_{T>0} = 2\int_0^{\infty} \frac{d^3p}{(2\pi\hbar)^3} f(p)$$

$$\rho^s_{T>0} = 2\int_0^{\infty} \frac{d^3p}{(2\pi\hbar)^3} \frac{m_N^*}{\sqrt{p^2 + m_N^{*\,2}}} f(p)$$

$$f_{\pm}(p) \equiv \frac{1}{1 + \exp\left[\left(\sqrt{p^2 + m^2} \mp \mu\right)/T\right]}$$

$$\rho = \int_0^{\infty} \frac{d^3p}{(2\pi\hbar)^3} f_{T,\mu,m}(p) \qquad \Rightarrow \boxed{\mu = \mu(\rho, T, m)}$$

$$E_{\rm kin} = \int_0^{\infty} \frac{d^3p}{(2\pi\hbar)^3} \left[\sqrt{p^2 + m^2} - m\right] f_{T,\mu,m}(p)$$

$$\Rightarrow \boxed{E_{\rm kin} = E_{\rm kin}(\rho, T, m)}$$

$$F = E - TS$$

$$\Omega = E - \mu N - TS$$

$$s \equiv S/V = -2\int_0^{\infty} \frac{d^3p}{(2\pi\hbar)^3} \left[f(p)\log(f(p)) + (1 - f(p))\log(1 - f(p))\right]$$

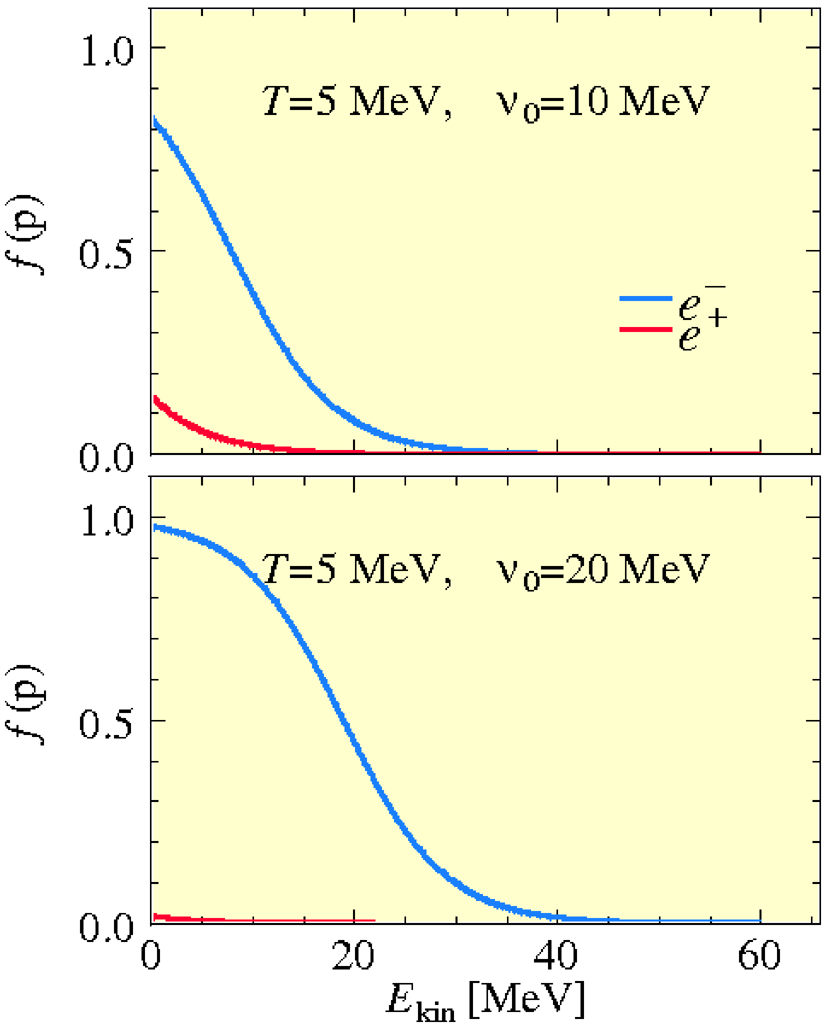
$$\Rightarrow \boxed{s = s(\rho, T, m)}$$

Store numerical values in a table.
Interpolate the discrete data when used.

Contribution of anti-particles is small.

We neglect anti-particles.

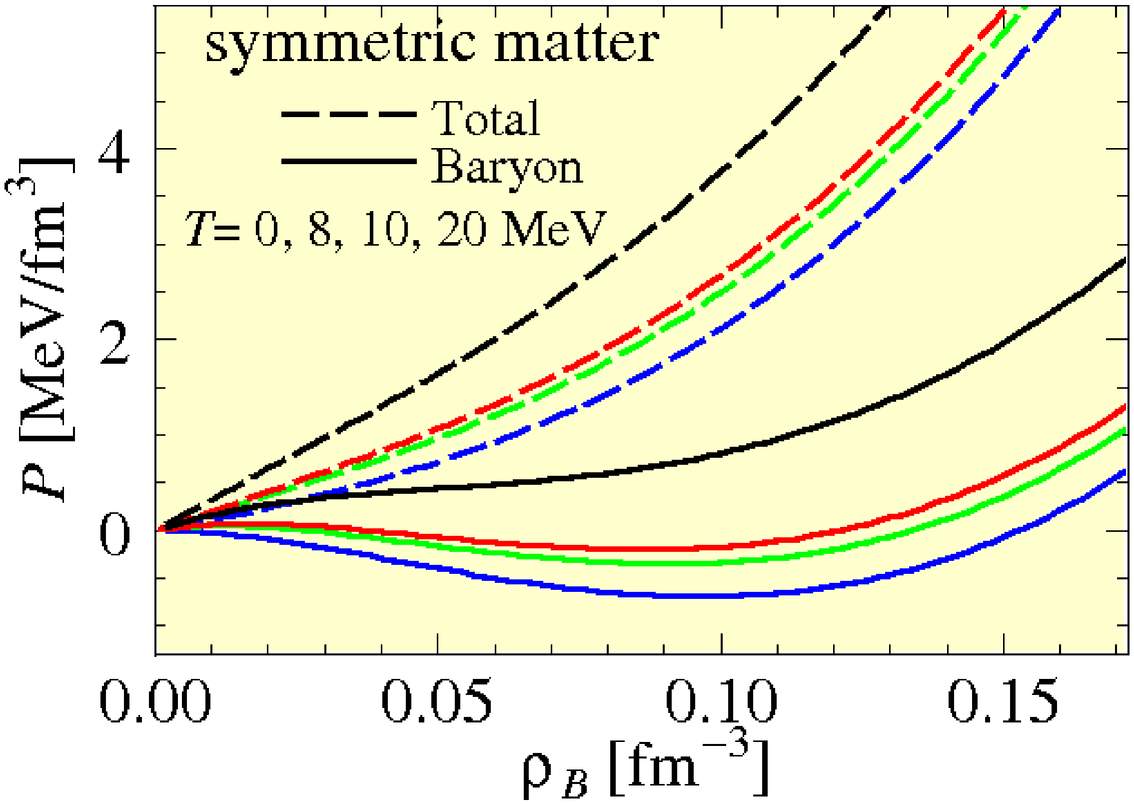
$$f_{\pm}(p) = \frac{1}{1+\exp\left[\left(\sqrt{p^2+m^2} \mp \mu_T\right)\Big/T\right]}$$



Pressure of uniform nuclear matter

• Total pressure is positive. Monotonically increases with density and temperature.

• Baryon partial pressure can be negative.



• Negative baryon pressure

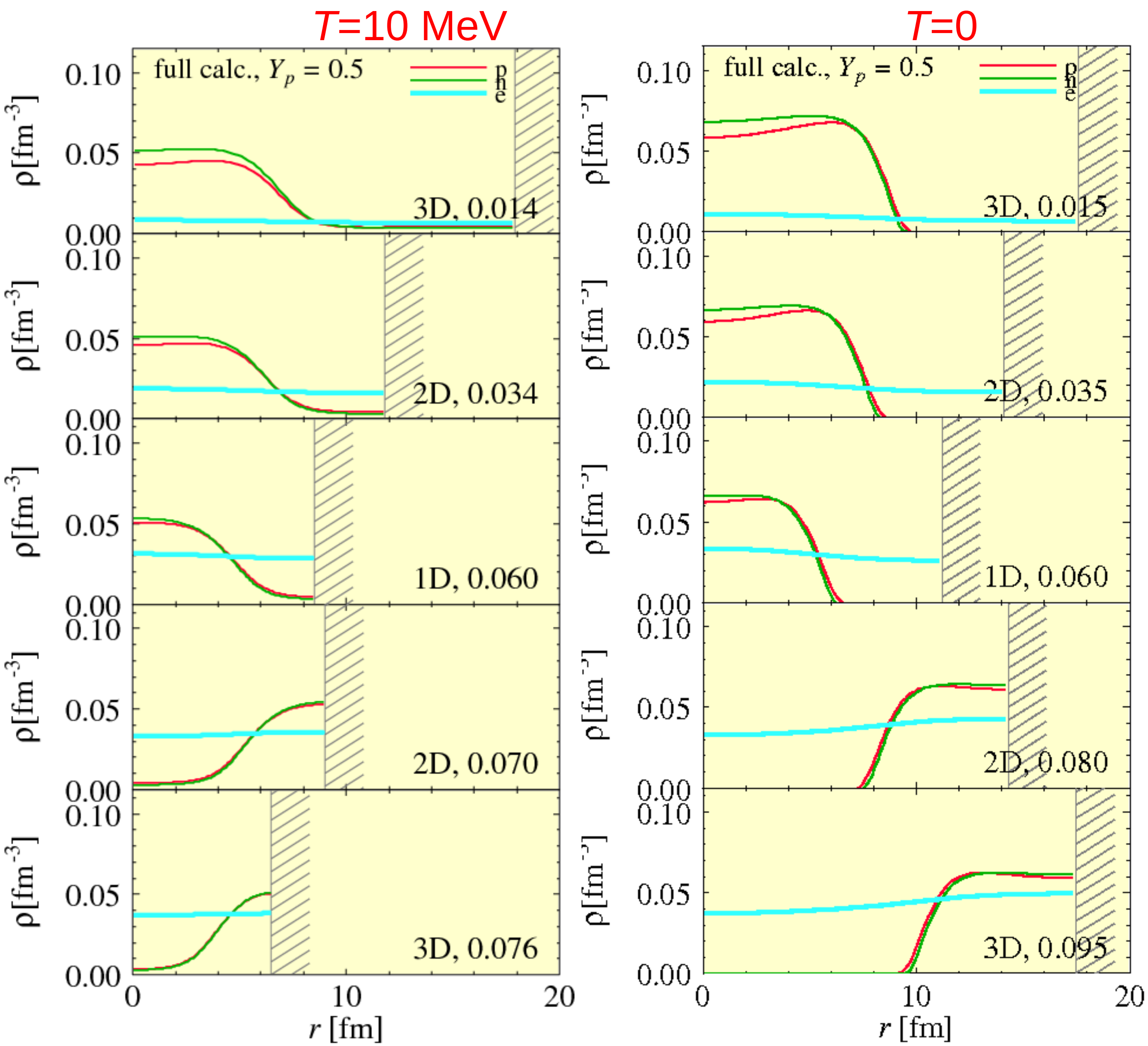
→ possibility of clustering.

Pasta structures at *T*=10 MeV

Mixed phase of liquid and gas.

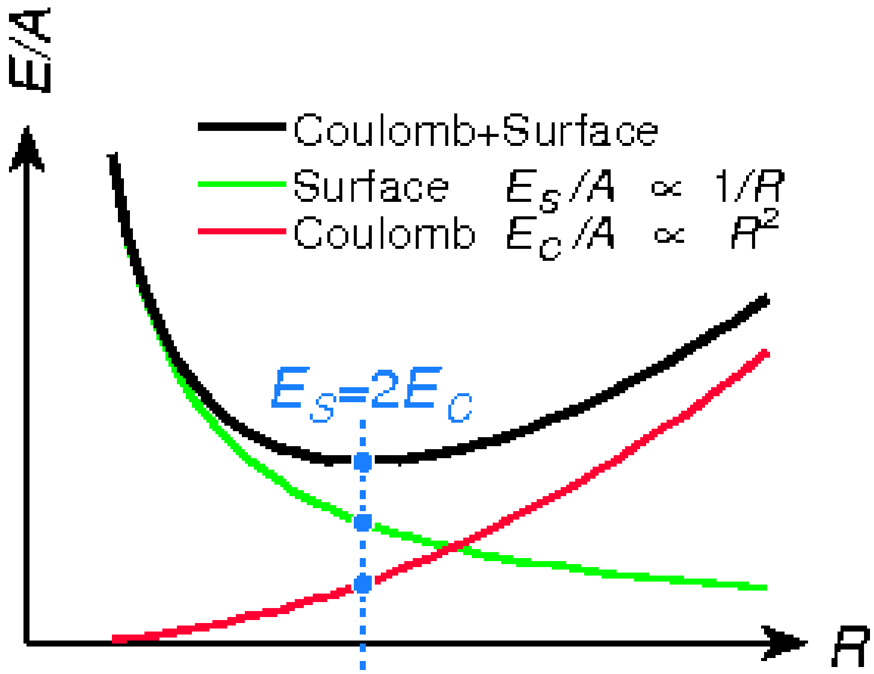
The surface is more vague than *T*=0 case.
→weaker surface tension
→ smaller size

Electron distribution is more uniform.
→less screening
→ smaller size



Why weaker surface tension and less screening lead to smaller structure size?

Dependence of *E/A* on *R*.



Weak surface tension and strong Coulomb

→ small *R*

Strong surface tension and weak Coulomb

→ large *R*

extreme case

→ no minimum.

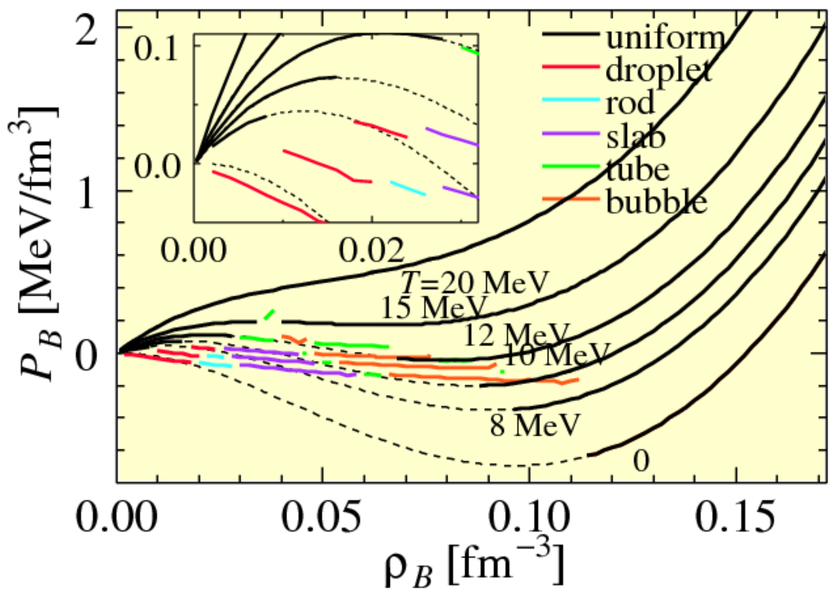
(pasta unstable)

EOS of matter

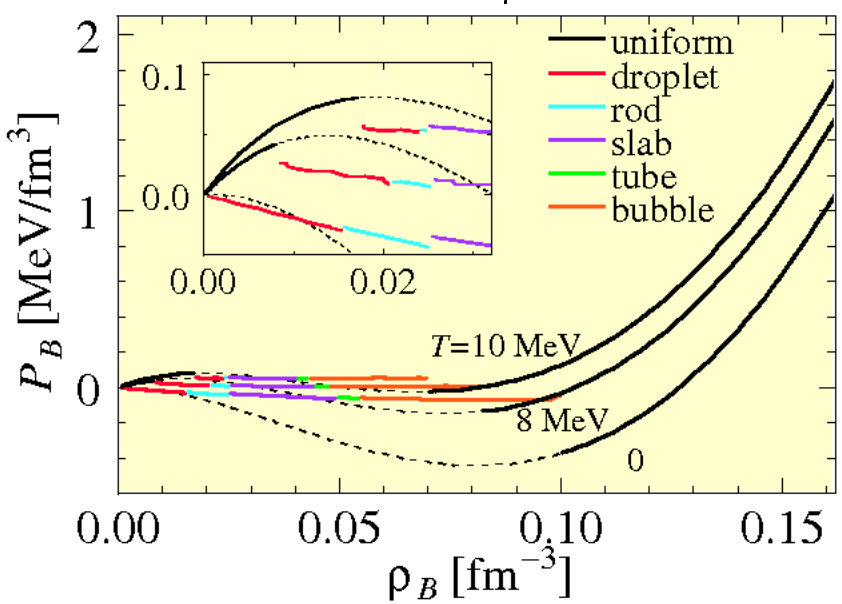
Choose free energy *F/A* minimum configurations
→ Pasta structures at T<15 MeV and uniform matter at T>15 MeV.

Positive $dP/d\rho$ at very low density.
(Similar to ideal gas?)
→ Uniform matter stable

Symmetric matter Y_p=0.5



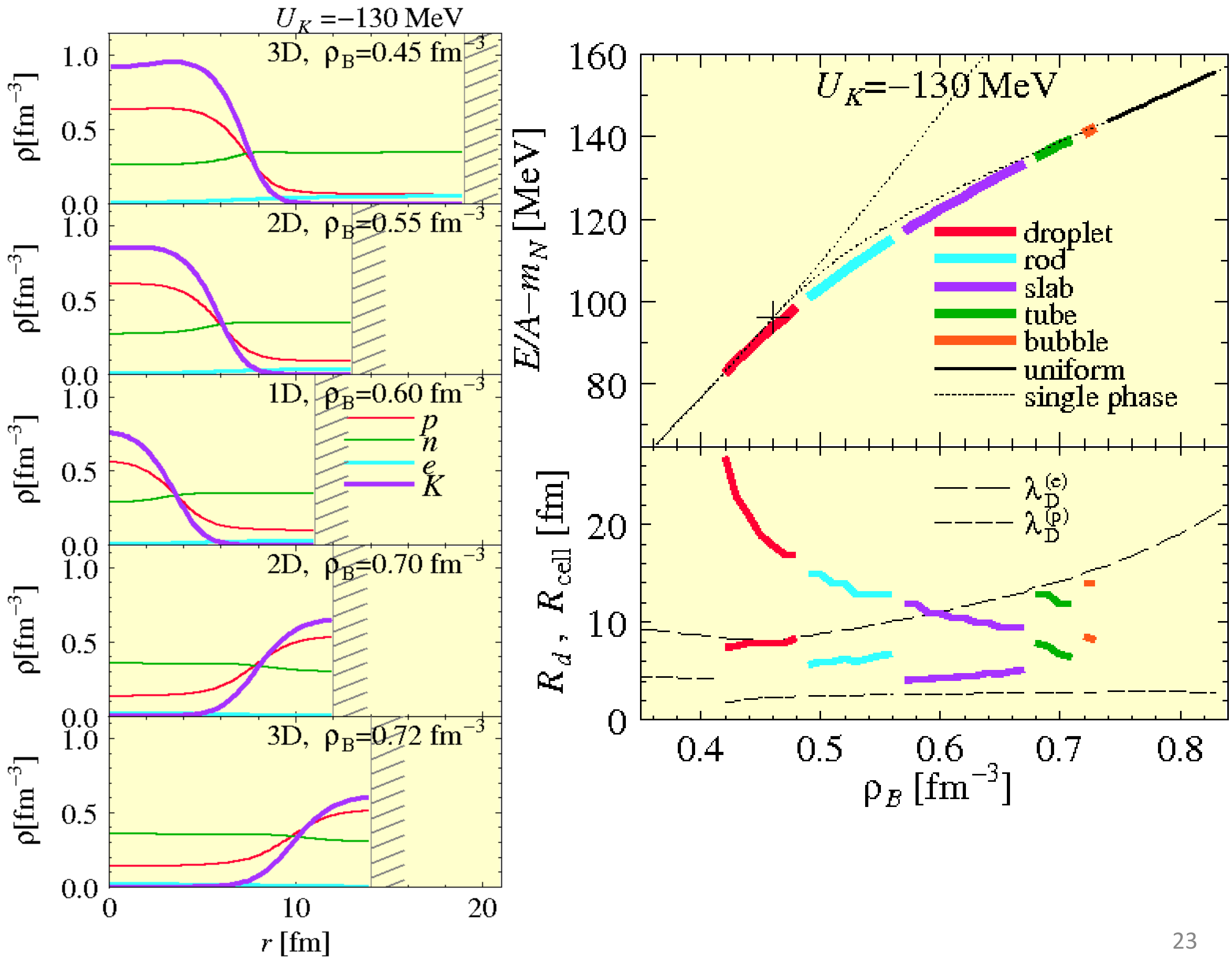
Asymmetric matter Y_p=0.3



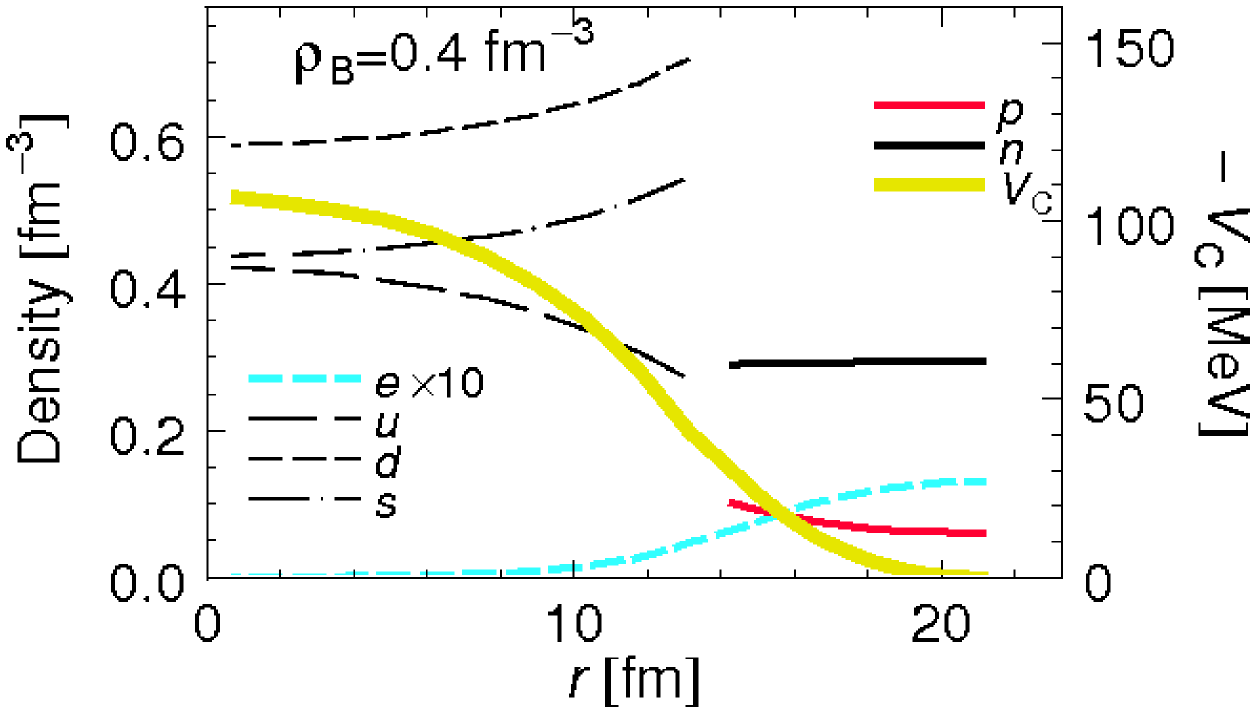
Mixed phase at higher densities

- Kaon condensation
  [PRC73(2006)035802,RRDP7(2006)1]
- Hadron-quark transition
  [PRD76(2007)123015,PLB659(2008)192]

Kaonic pasta structure



Hadron-quark droplet



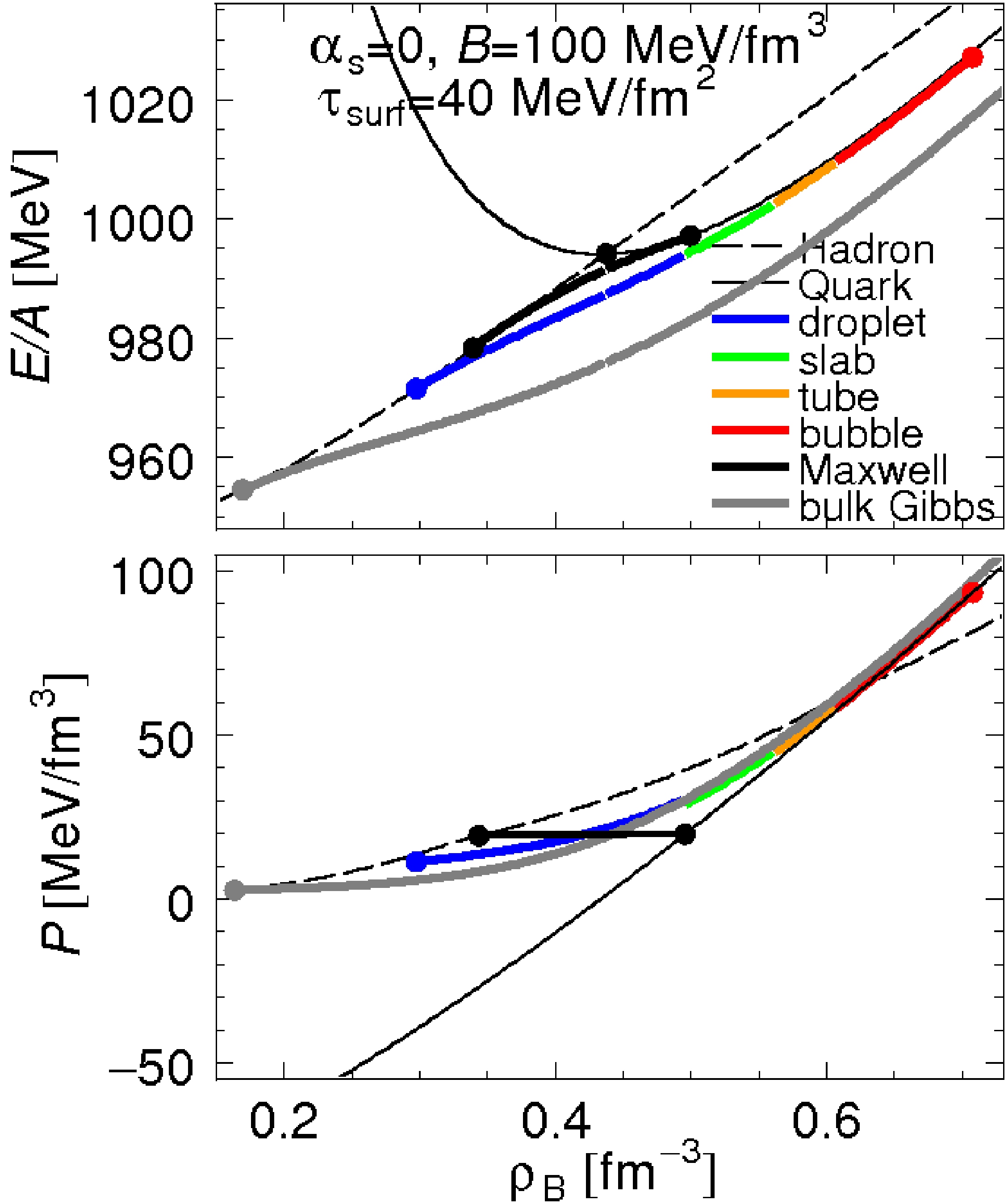
Quark phase is negatively charged.

→ *u* quarks are attracted and *ds* quarks repelled.

Same thing happens to *p* in the hadron phase.

EOS of matter

Full calculation is close to the Maxwell construction (local charge neutral). Far from the bulk Gibbs calculation (neglects the surface and Coulomb).



Summary

- We have studied ``Pasta'' structures of low density matter using RMF model for T=0 and T>0.
- Negative baryon partial pressure causes the clustering of matter → formation of pasta.
- At finite *T,* less screening and weaker surface tension reduce the structure size.
- At finite *T*, "liquid-gas" mixed phase appears between uniform "gas" and uniform "liquid".
- At higher densities, other kinds of pasta structures appear as structured mixed phases.